Vang

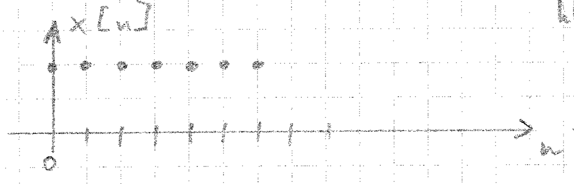
2.2a) $h[n] = \alpha^{n} \cdot \sigma[n]$

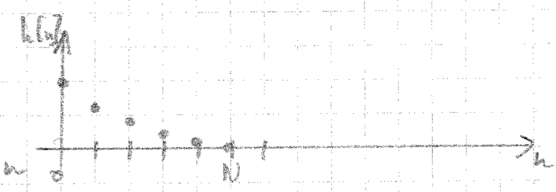
$x[n] = \sigma[n]$

$y[n] = ?$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} h[k] \cdot x[n-k]$$





$$y[n] = \sum_{k=0}^{N} \alpha^{k} \cdot \sigma[k] \cdot \sigma[n-k] = \sigma[n] \cdot \sum_{k=0}^{N} \alpha^{k}$$

$$= \frac{1 - \alpha^{N+1}}{1 - \alpha} \cdot \sigma[n]$$

2.4) $h[n] = h_1[n] * (h_2[n] - (h_3[n] * h_4[n]))$

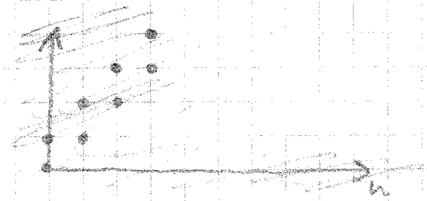
$h_3[n] * h_4[n] = h_2[n-1]$

Faltung mit $\delta[n-m]$, ergibt Verschiebung um m!

$h_3[n] = h_2[n]$ ist und $h_4[n] = \delta[n-1] \Rightarrow h_2[n-1]$!

$$\{h_2 - (h_3 * h_4)\}[n] = (h_2 - h_2[n-1]) = (n+1) \cdot \sigma[n] - n \cdot \sigma[n-1]$$

$= \sigma[n]$



$$h[n] = h_1[n] * \sigma[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{k} \cdot \sigma[k] \cdot \sigma[n-k]$$

$$= \sum_{k=0}^{N} \left(\frac{1}{2}\right)^{k} \cdot \sigma[n] = \frac{1 - \left(\frac{1}{2}\right)^{N-1}}{1 - \frac{1}{2}}$$

$2.3)$ $y[n] = ?$ Vanc

$h[n] = h_1[n] * h_2[n] \Rightarrow y[n] = x[n] * h[n]$

oder

$y_1[n] = x[n] * h_1[n]$

$y[n] = y_1[n] * h_2[n]$

LTI verdauschen! $\Rightarrow$ $y_1[n] * x[n] * h_2[n]$

$y_1[n] = \alpha^n \cdot \sigma[n] - \alpha \cdot \alpha^{n-1} \cdot \sigma[n-1] = \alpha^n \cdot \underbrace{\{\sigma[n] - \sigma[n-1]\}}_{\delta[n]}$

$= \alpha^n \cdot \delta[n] =$

$y[n] = y_1[n] * h_2[n] = \left(\frac{1}{2}\right)^n \cdot \left\{\sin \frac{\pi}{10} n\right\}^2$

$1.3)$ a) $x[n] = 1 - \cos\frac{\pi}{4} n$ mit Tabelle! $1 \hat{=} e^{j\frac{2\pi}{N} n \cdot 0}$

$c_k = \delta[k] - \frac{1}{2}\delta[k-1] + \frac{1}{2}\delta[k+1]$

c) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-3k]$

$c_k = \frac{1}{3}$

~~d)~~ $\cancel{x[n] = e^{j\frac{2\pi l}{N} n}}$

$\cancel{c_k = \delta[k-l]}$

$3.1)$ b) $x[n] = 2^n \cdot \sigma[n]$

$X(e^{j\Theta}) = \frac{1}{1 - \frac{1}{2} \cdot e^{-j\Theta}}$

4.5) a) $y[n] + 3y[n-1] = x[n]$ Vaus

$Y(z) + 3z^{-1}Y(z) + 3 = X(z)$

$Y(z) + 3z^{-1}Y(z) = X(z) - 3$

$Y(z) \cdot (1 + 3z^{-1}) = X(z) - 3 \quad | : (1 + 3z^{-1})$

$Y(z) = \left(\frac{z}{z - \frac{1}{2}} - 3\right) \cdot \frac{1}{1 - 3z^{-1}} = \frac{z}{z+3} \cdot \left(\frac{z}{z - \frac{1}{2}} - 3\right)$

$= z \cdot \underbrace{\frac{z}{(z+3)\cdot(z-\frac{1}{2})}} - 3z \frac{1}{z+3} =$

$\hookrightarrow \quad \frac{r_1}{z+3} + \frac{r_2}{z - \frac{1}{2}} \Rightarrow \quad r_1 = \frac{6}{7}, \quad r_2 = \frac{1}{7}$

$Y(z) = \frac{6}{7} \frac{z}{z+3} + \frac{1}{7} \frac{z}{z - \frac{1}{2}} - 3 \frac{z}{z+3}$

$y[n] = \frac{6}{7}(-3)^n \cdot \sigma[n] + \frac{1}{7}\left(\frac{1}{2}\right)^n \sigma[n] - 3(-3)^n \cdot \sigma[n]$

b) und c) genau so!

~~2.1a) $y[n] = x_a[n] - x[n-1]$~~

~~Linearität: $a\, y_1[n] + b \cdot y_2[n] \Leftarrow a x_1[n] + b x_2[n]$~~

~~$y_1[n] = a x_1[n] + b x_1[n-1]$~~

~~$y_2[n] = a x_2[n] +$~~

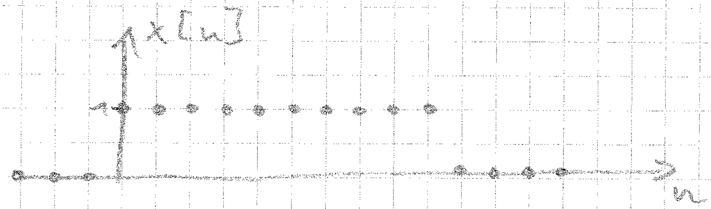
4.1 e) $x[n] = \left(\frac{1}{2}\right)^{n} \cdot \sigma[-n] = 2^{-n} \cdot \sigma[-n]$ Vaus

$X(z) = \frac{z^{-1}}{z^{-1} - 2}$ — u —

f) $x[n] = n \cdot e^{-\alpha n} \cdot \sigma[n]$

$X(z) = -z \frac{d}{dz}\left(\frac{z}{z - e^{-\alpha}}\right) = \frac{z \cdot e^{-\alpha}}{(z - e^{-\alpha})^2}$

g)



$x[n] = \sigma[n] - \sigma[n-10]$

$X(z) = \frac{z}{z-1} - z^{-10} \cdot \frac{z}{z-1}$ trivial

4.3) a) $y[n] - \frac{1}{2}\, y[n-1] + \frac{1}{4} \cdot y[n-2] = x[n]$

$H(z) = \frac{Y(z)}{X(z)}$

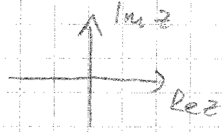
$Y(z) - \frac{1}{2} z^{-1} Y(z) + \frac{1}{4} z^{-2} Y(z) = X(z)$

$Y(z) \cdot \left(1 - \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}\right) = X(z)$

$\Rightarrow \; H(z) = \frac{1}{1 - \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}}$

$= \frac{4z^2}{z^2 - \frac{1}{2} z + \frac{1}{4}}$

b) trivial $N(z) = 0$ $P(z) = 0 \Rightarrow$



c) $Y(z) = H(z) \cdot X(z)$ $X(z) = \frac{z}{z - \frac{1}{2}}$

Partialbruchzerlegung

$\Rightarrow y[n]$

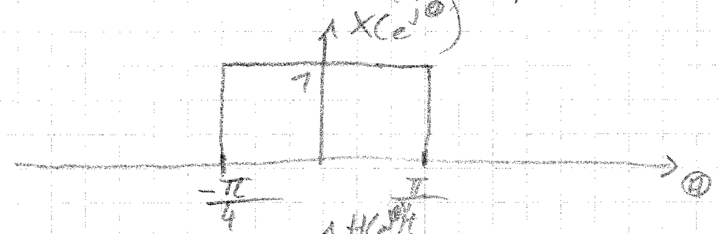
3.3) c) $x[n] = \delta[n+1] + \delta[n-1]$

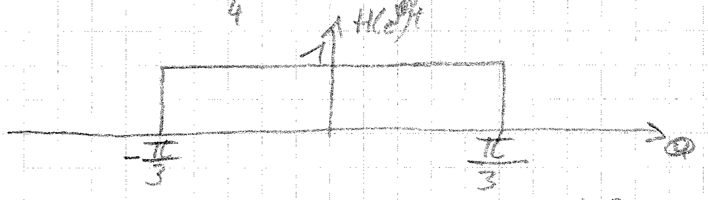
$$y[n] = x[n] * h[n] = \frac{\sin\frac{\pi}{3}(n+1)}{(n+1)\cdot\pi} + \frac{\sin\frac{\pi}{3}(n-1)}{(n-1)\cdot\pi}$$

d) $x[n] = \frac{\sin\frac{\pi}{4}\cdot n}{n\cdot\pi}$

Vang

$$X(e^{j\Omega}) = \begin{cases} 1 & 0 \le |\Omega| < \frac{\pi}{4} \\ 0 & \frac{\pi}{4} \le |\Omega|\ \pi \end{cases}$$





$$Y(e^{j\Omega}) = X(e^{j\Omega})\, H(e^{j\Omega}) = X(e^{j\Omega})$$

$$y[n] = \frac{\sin\frac{\pi}{4}n}{\pi n}$$

4.2) a) $X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{z}{z-\frac{1}{2}}$

$\circ\!\!-\!\!\bullet$ $x_R[n] = \alpha^n \cdot \sigma[n] = \left(\frac{1}{2}\right)^n \cdot \sigma[n]$ $\quad |z| > |\alpha|$

$x_L[n] = -\alpha^n \cdot \sigma[-n-1] = -\left(\frac{1}{2}\right)^n \cdot \sigma[-n-1]$ $\quad |z| < |\alpha|$

4.1. a) $x[n] = \delta[n-1]$ Diagramm Aurial

$X(z) = z^{-1}$.

b) $x[n] = \delta[n+1]$ — " —

$X(z) = z$

c) $x[n] = \delta[n-1] + \delta[n+1]$

$X(z) = z^{-1} + z = \frac{1+z^2}{z}$ — " —

d) $x[n] = \left(\frac{1}{2}\right)^n \cdot \sigma[n]$

$X(z) = \frac{z}{z-\frac{1}{2}}$ — " —

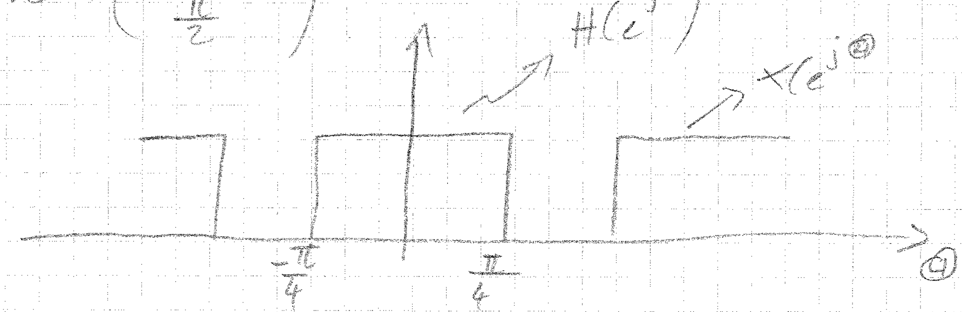
3.3) e) $x[n] = \frac{\sin\frac{\pi}{4}n}{\pi n} \cdot \cos\frac{3}{4}\pi n$

$$= \underbrace{\frac{\sin\frac{\pi}{4}n}{\pi n}}_{x_2[n]} \cdot \underbrace{\frac{1}{2}\left(e^{j\frac{3\pi}{4}n} + e^{-j\frac{3}{4}\pi n}\right)}_{x_1[n]}$$

$$\circ\!-\!\bullet \quad X_1(e^{j\Omega}) = \frac{1}{2} \cdot 2\pi \cdot \left(\delta\left(\Omega - \frac{3\pi}{4}\right) + \delta\left(\Omega + \frac{3\pi}{4}\right)\right)$$

$$X(e^{j\Omega}) = \frac{1}{2\pi}(X_2 * X_1)(e^{j\Omega})$$

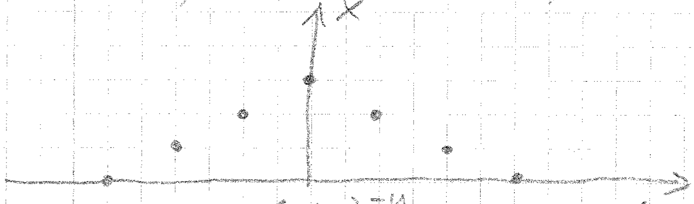
$$= \frac{1}{2} \cdot 2\pi \cdot \frac{1}{2\pi} \operatorname{rect}\left(\frac{\Omega}{\frac{\pi}{2}}\right) * X_1 = \frac{1}{2}\operatorname{rect}\left(\frac{\Omega - \frac{3\pi}{4}}{\frac{\pi}{2}}\right) + \frac{1}{2}\operatorname{rect}\left(\frac{\Omega + \frac{3\pi}{4}}{\frac{\pi}{2}}\right)$$



$$\Rightarrow \quad Y(e^{j\Omega}) = X(e^{j\Omega}) \cdot H(e^{j\Omega}) = 0$$

$$y[n] = 0$$

3.1 d) $x[n] = \left(\frac{1}{2}\right)^{|n|}$



$$\Rightarrow x[n] = \left(\frac{1}{2}\right)^{-n} \cdot \sigma[n] + \left(\frac{1}{2}\right)^{n} \sigma[n] - \delta[n]$$

$$\circ\!-\!\bullet \quad X(e^{j\Omega}) = \frac{1}{1 - \frac{1}{2} \cdot e^{+j\Omega}} + \frac{1}{1 + \frac{1}{2}e^{-j\Omega}} - 1$$

$$= \frac{3}{5 - 4\cos\Omega}$$

e) $x[n] = n \cdot \left(\frac{1}{2}\right)^{|n|} \quad \circ\!-\!\bullet \quad X(e^{j\Omega}) = j\frac{d X_1(e^{j\Omega})}{d\Omega}$

$$X_1(e^{j\Omega}) = \frac{3}{5 - 4\cos\Omega}$$

$$X(e^{j\Omega}) = -\frac{12 j \sin\Omega}{(5 - 4\cos\Omega)^2}$$

2.8) a) $Y(e^{j\Theta}) = j\,\frac{dX(e^{j\Theta})}{d\Theta}$

$y[n] = n \cdot x[n]$

$b_1)$ $y[n] = n \cdot \delta[n - N_0]$

$b_2)$ $y[n] = n \cdot \sigma[n]$

$b_3)$ $y[n] = n \cdot \frac{\sin \Theta_0 n}{\pi n}$

4.10) I $\quad y[n] + \frac{1}{2} y[n-1] + w[n] + w[n-1] = \frac{1}{2} x[n]$

II $\quad y[n] - \frac{1}{2} y[n-1] + 2w[n] - w[n-1] = \frac{1}{2} x[n]$

a) $H(e^{j\Theta})$ und $h[n]$

$$z = e^{j\Theta} \Rightarrow H(e^{j\Theta}) = H(z)$$

$H(z) = \frac{Y(z)}{X(z)} \Rightarrow W(z)$ ~~dann~~ durch $Y(z)$ ausdrücken!

$$\mathrm{I} + \mathrm{II} \Rightarrow 2y[n] + 3w[n] = 0 \circ\!\!-\!\!\bullet\ 2Y(z) + 3W(z) = 0$$

$$W(z) = -\frac{2}{3} Y(z)$$

$$\mathrm{I} \bullet\!\!-\!\!\circ\ \mathrm{I} \Rightarrow Y(z) + \frac{1}{2} Y(z) \cdot z^{-1} - \frac{2}{3} Y(z) - \frac{2}{3} Y(z) \cdot z^{-1} = \frac{1}{2} X(z)$$

$$Y(z) \cdot \left(1 + \frac{1}{2} z^{-1} - \frac{2}{3} - \frac{2}{3} z^{-1}\right) = \frac{1}{2} X(z)$$

$$Y(z) \cdot \left(\frac{1}{3} - \frac{1}{6} z^{-1}\right)$$

$$\Rightarrow H(z) = \frac{1}{2} \cdot \left(\frac{1}{3} - \frac{1}{6} z^{-1}\right)^{-1} = \frac{1}{2} \cdot \left(\frac{2z-1}{6z}\right)^{-1} = \frac{1}{\cancel{2}} \cdot \frac{{}^{3}\cancel{6}z}{2z-1}$$

$$\Rightarrow H(z) = \frac{3}{2} \cdot \frac{z}{z - \frac{1}{2}}$$

$$\bullet\!\!-\!\!\circ\ h[n] = \frac{3}{2} \cdot \left(\frac{1}{2}\right)^n \cdot \sigma[n]$$

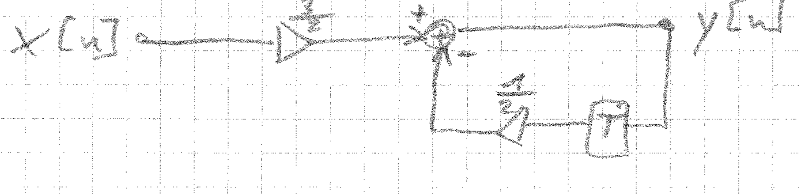
b) $Y(z) = H(z) \cdot X(z) = \frac{3}{2} \cdot \frac{z}{z - \frac{1}{2}} \Big/ z^{-1} \cdot X(z) = \frac{3}{2} \cdot \frac{1}{1 - \frac{1}{2} z^{-1}} X(z)$

$$Y(z) = \frac{3}{2} X(z) \cdot \frac{1}{1 - \frac{1}{2} z^{-1}} \quad \Big| \cdot (\ )$$

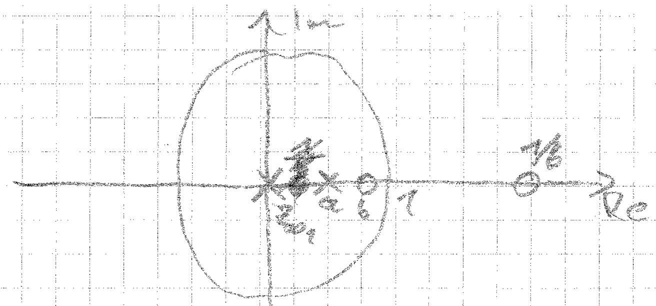
$$\frac{3}{2} X(z) = Y(z) - \frac{1}{2} Y(z) \cdot z^{-1}$$

$$\bullet\!\!-\!\!\circ\ \frac{3}{2} x[n] = y[n] - \frac{1}{2} y[n-1]$$

c) aus b) folgt



4.9) a)



$0 < a < 1$

$0 < b < 1$

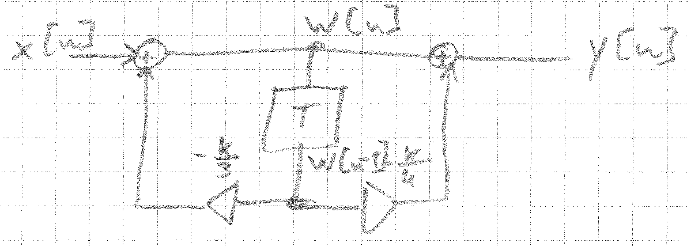
$a \neq b$

b) $H(z) = k \frac{(z - z_{01}) \cdot (z - z_{02})}{(z - z_{\infty 1}) \cdot (z - z_{\infty 2})} = \frac{(z - b) \cdot (z - \frac{1}{b})}{z \cdot (z - a)} \cdot k$

k ... Verstärkungsfaktor!

$H(z) = 1 \quad z = 1 \Rightarrow \quad 1 = \frac{(1 - b) \cdot (1 - \frac{1}{b})}{1 - a} \cdot k$

4.4)



$\Rightarrow \quad w[n] = x[n] - \frac{k}{3} w[n-1]$

$y[n] = w[n] - \frac{k}{4} w[n-1]$

a) $\longleftrightarrow \quad W(z) = X(z) - \frac{k}{3} z^{-1} \cdot W(z)$

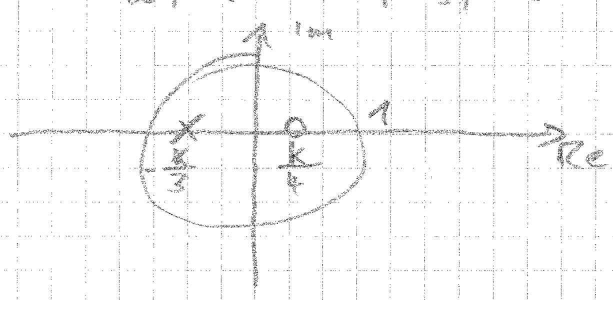
$Y(z) = W(z) - \frac{k}{4} z^{-1} W(z)$

$$H(z) = \frac{W(z) \cdot (1 - \frac{k}{4} z^{-1})}{W(z) \cdot (1 + \frac{k}{3} z^{-1})} = \frac{\frac{4z - k}{4z}}{\frac{3z + k}{3z}} = \underline{\underline{\frac{z - \frac{k}{4}}{z + \frac{k}{3}}}}$$

b) $z_0 = \frac{k}{4} \qquad z_\infty = -\frac{k}{3}$

damit Filter stabil muß Pol im EK liegen!

$|z_\infty| < 1 \Rightarrow \left|-\frac{k}{3}\right| < 1 \Rightarrow \underline{\underline{|k| < 3}}$



4.4) a) $h[n] = ?$

$$H(z) = \frac{z - \frac{k}{4}}{z + \frac{k}{3}} = \frac{z}{z + \frac{k}{3}} - \frac{k}{4} \cdot \frac{1}{z + \frac{k}{3}}$$

$$= \frac{z}{z + \frac{k}{3}} - \frac{z}{z + \frac{k}{3}} z^{-1} \cdot \frac{k}{4}$$

z^{-1} damit man $\frac{z}{z-a}$ hat!

$$h[n] = \left(-\frac{k}{3}\right)^n \cdot \sigma[n] - \left(-\frac{k}{3}\right)^{n-1} \cdot \sigma[n-1] \cdot \frac{k}{4} \checkmark$$

$a[n] = h[n] * \sigma[n]$

$\circ\!\!-\!\!\bullet \; A(z) = H(z) \cdot \frac{z}{z-1} = \ldots$

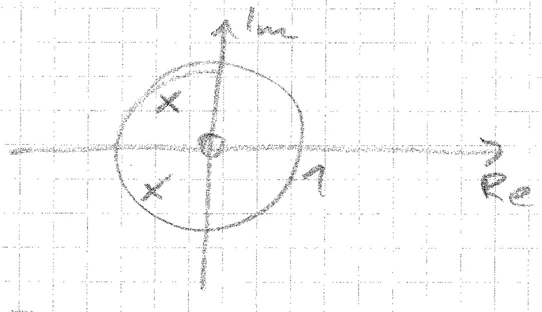
4.7) a) $H(z) = \frac{Y(z)}{X(z)}$

$$Y(z) \cdot \left(1 + z^{-1} + \frac{1}{2} z^{-2}\right) = \frac{5}{2} X(z)$$

$$\Rightarrow H(z) = \frac{5}{2} \cdot \frac{1}{1 + z^{-1} + \frac{1}{2} z^{-2}} \Big/ z^2 = \underline{\underline{\frac{5}{2} \cdot \frac{z^2}{z^2 + z + \frac{1}{2}}}}$$

b) $z_0 = 0$

$z_{\infty 1,2} = -\frac{1}{2} \pm j\frac{1}{2}$

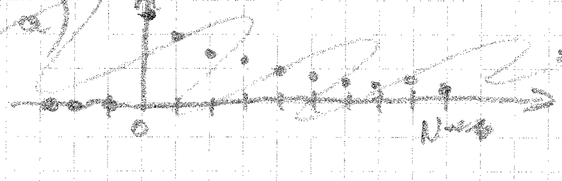


c) $a[n] = h[n] * \sigma[n]$

$\circ\!\!-\!\!\bullet \; A(z) = \frac{5}{2} \cdot \frac{z^2}{z^2 + z + \frac{1}{2}} \cdot \frac{z}{z-1} = \ldots$

$\cancel{= \frac{5}{2} + z^{-1} \cdot \frac{z}{z-1} + \ldots}$

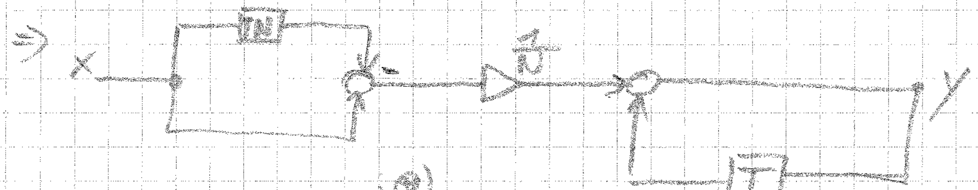
~~4.8) a)~~



$\cancel{\Rightarrow 2^n \cdot (\sigma[n] - \sigma[n-N])}$

2.7) a) $y[n] = y[n-1] + \frac{1}{N} \cdot (x[n] - x[n-N])$

$\Rightarrow$



b) $H(e^{j\Theta}) = \frac{Y(e^{j\Theta})}{X(e^{j\Theta})}$ ⚬—⚬ $h[n]$

$$Y(e^{j\Theta}) = Y(e^{j\Theta}) \cdot e^{-j\Theta} + \frac{1}{N} \cdot \left(X(e^{j\Theta}) - e^{j\Theta N} \cdot X(e^{j\Theta})\right)$$

$$\Rightarrow H(e^{j\Theta}) = \frac{1}{N} \cdot \frac{1 - e^{j\Theta N}}{1 - e^{-j\Theta}} = \frac{1}{N} \cdot \left\{ \frac{1}{1 - e^{-j\Theta}} - e^{j\Theta N} \cdot \frac{1}{1 - e^{-j\Theta}} \right\}$$

Lsg: mit Trafoeigenschaften und Tabellen!

$$●\!\!-\!\!\circ \quad h[n] = \frac{1}{N} \cdot \{\sigma[n] - \sigma[n-N]\}$$

$$a[n] = h[n] * \sigma[n]$$

2.1) $y[n] = \sum_{n-1}^{n+1} (x[k+1] - x[k] + x[k-1])$

$= x[n-1+1] - x[n-1] + x[n-1-1] + x[n+1] - x[n]$

$+ x[n-1] + x[n+1+1] - x[n+1] + x[n+1-1]$

$= x[n] + x[n-2] + x[n+2]$

b) $y[n] = x[n] + x[n-2] + x[n+2] \Rightarrow h[n] = \delta[n] + \delta[n+2] + \delta[n-2]$

$$\circ\!\!-\!\!\circ \quad H(e^{j\Theta}) = 1 + \underbrace{e^{j2\Theta} + e^{-j2\Theta}}_{2 \cdot \cos(2\Theta)} = \underline{1 + 2\cos 2\Theta}$$

c) $x[n] = (-1)^n$

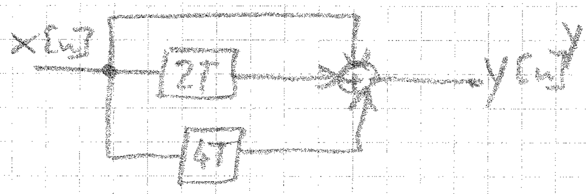
$$y[n] = (-1)^n + (-1)^{n+2} + (-1)^{n-2} = \underline{3 \cdot (-1)^n}$$

d) $x[n] = \lambda^n$ ges: λ für $y[n] = 0$

$$0 = \lambda^n + \lambda^{n+2} + \lambda^{n-2} \Rightarrow \lambda^n \cdot (\lambda^2 + \lambda^{-2} + 1) = 0$$

$$\underline{\lambda^n = 0} \Rightarrow \lambda^4 + \lambda^2 + 1 = 0 \Rightarrow \lambda_{4,3,2,1} = \pm\sqrt{-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}}$$

2.9 e)



$y[n] = x[n] + x[n-1]$
$+ x[n-2]$