Liebe Leute!

Viele dieser Beispiele stammen aus den alten getrennten Prüfungen aus TET1 und TET2. Sie haben trotzdem weder für die gesamt TET- noch für die neuen getrennten TET-Prüfungen nichts an Aktualität eingebüßt.

Wenn Du zu Deiner Prüfung neue Beispiele bekommst, arbeite sie bitte aus (oder auch nicht) und bring sie uns vorbei, damit wir diese Beispielsammlung erweitern können.

Danke!

FET

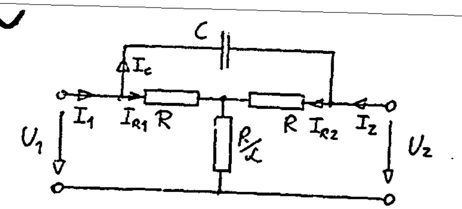
Inhalt:

1. Teil

2. Teil

1.1) Geg: überbrücktes T-Glied

Ges: $G(j\omega) = \frac{U_2(j\omega)}{U_1(j\omega)}$



① $U_1 = I_{R1} \cdot R + (I_{R1} + I_{R2}) \cdot \frac{R}{\alpha}$

② $U_2 = I_{R2} \cdot R + (I_{R1} + I_{R2}) \cdot \frac{R}{\alpha}$

③ $I_2 = 0!$ für Übertragungsfkt.

④ $I_1 = I_{R1} + I_C$

⑤ $I_2 + I_C = I_{R2}$ mit ③ $\Rightarrow I_C = I_{R2}$

⑥ $I_C = j\omega C(U_1 - U_2)$ $\Rightarrow I_{R2} = j\omega C(U_1 - U_2)$ ⑥'

①,②,⑥' $\Rightarrow I_{R1}$ und I_{R2} eliminieren $\Rightarrow G(j\omega)$

⑥ in ①: $U_1 = I_{R1} \cdot R(1 + 1/\alpha) + \frac{j\omega CR}{\alpha}(U_1 - U_2)$

⑥ in ②: $U_2 = I_{R1} \cdot R/\alpha + R(1 + 1/\alpha) \cdot j\omega C(U_1 - U_2)$

$I_{R1} = \frac{\alpha}{R}\left[U_2(1 + R(1+1/\alpha) j\omega C) - U_1 R(1+1/\alpha) j\omega C\right]$

② in ①: $U_1 = U_2\left[R(1+1/\alpha) \cdot \frac{\alpha}{R} \cdot (1 + j\omega RC(1+1/\alpha)) + \frac{j\omega RC}{\alpha}(-1)\right]$

$-U_1\left[R(1+1/\alpha) \cdot \frac{\alpha}{R} \cdot R(1+1/\alpha) j\omega C - j\omega C \frac{R}{\alpha}\right]$

$U_1\left[1 + j\omega RC(\alpha(1+1/\alpha)^2 - 1/\alpha)\right] = U_2\left[(1+\alpha) + j\omega RC[(1+\alpha)(1+1/\alpha) - 1/\alpha]\right]$

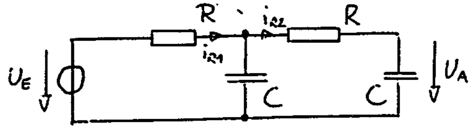
$$\underline{\underline{\frac{U_2}{U_1}}} = \frac{1 + j\omega RC(\alpha + 2 + 1/\alpha - 1/\alpha)}{1 + \alpha + j\omega RC(1 + \alpha + 1/\alpha + 1 - 1/\alpha)} = \underline{\underline{\frac{1 + j\omega RC(2+\alpha)}{1 + \alpha + j\omega RC(2+\alpha)}}}$$

Proben: $\alpha = 0 \Rightarrow \frac{U_2}{U_1} = \frac{1 + j2\omega RC}{1 + j2\omega RC} = 1$ ✓

$\alpha = \infty \Rightarrow \frac{U_2}{U_1} = \frac{1/\alpha + j\omega RC(2/\alpha + 1)}{1/\alpha + 1 + j\omega RC(2/\alpha + 1)} = \frac{j\omega RC}{1 + j\omega RC} = \frac{R}{R + \frac{1}{j\omega C}}$ ✓

Pole: $j\omega \to s \Rightarrow 1 + \alpha + sRC(2+\alpha) = 0 \Rightarrow s = -\frac{1+\alpha}{RC(2+\alpha)} < 0$ für $\alpha > -1$

eg: In der dargestellten Schaltung ist u_E die Eingangsgröße und u_A die Ausgangsgröße. Bestimmen Sie die zugehörige Übertragungsfunktion $G(s)$. (Bezug: $U_{EB}=U_{AB}$, $T_B=RC$)



$= U_{EB} = U_{AB} \rightarrow u = \frac{U_E}{U_B} \qquad y = \frac{U_A}{U_B}$

$= R\cdot C \qquad \tau = \frac{t}{T_B} = \frac{t}{RC} \qquad dt = R\cdot C\cdot d\tau \;; dt^2 = (R\cdot C)^2 d\tau^2$

$u_E = R\cdot i_{R1} + u_{C1}$

$u_{C1} = U_A + i_{R2}\cdot R$

$i_{R1} = i_{C1} + i_{R2}$

$i_{C1} = C\frac{du_{C1}}{dt}$

$i_{R2} = C\frac{dU_A}{dt}$

② in ① $\quad u_E = Ri_{C1} + Ri_{R2} + U_A + Ri_{R2} = U_A + R(i_{C1} + 2i_{R2})$

in ④ $\quad i_{C1} = C\frac{d}{dt}(U_A + i_{R2}\cdot R) = C\frac{dU_A}{dt} + RC\frac{di_{R2}}{dt} = C\frac{dU_A}{dt} + RC^2\frac{d^2U_A}{dt^2}$ mit ⑤

④ in ① $\quad u_E = U_A + RC\frac{dU_A}{dt} + R^2C^2\frac{d^2U_A}{dt^2} + 2RC\frac{dU_A}{dt}$

$u\cdot U_B = y\cdot U_B + U_B\cdot 3\,\frac{dy}{dt} + U_B\frac{d^2y}{d\tau^2}$

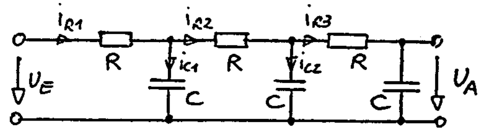
$\Rightarrow \quad \frac{d^2y}{d\tau^2} + 3\frac{dy}{d\tau} + y = u \quad / \mathcal{L} \qquad y \circ\!\!-\!\!\bullet\, Y(s) \quad \frac{dy}{d\tau} = sY(s)$

$s^2Y(s) + 3sY(s) + Y(s) = U(s) \qquad \frac{d^2y}{d\tau^2} \circ\!\!-\!\!\bullet\, s^2Y(s)$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^2 + 3s + 1}$$

FET

13) Geg: TET1:



Geg: $G(s)$

$U_B = U_{BE} = U_{BA} \;; T_B = RC$

$\tau = \frac{t}{RC}$

$d\tau = \frac{dt}{RC}, \; d\tau^2 = \left(\frac{dt}{RC}\right)^2, \; d\tau^3 = \left(\frac{dt}{RC}\right)^3$

① $U_E = i_{R1}\cdot R + U_{C1}$

② $U_{C1} = i_{R2}\cdot R + U_{C2}$

③ $U_{C2} = i_{R3}\cdot R + U_A$

④ $i_{R1} = i_{R2} + i_{C1} \Rightarrow i_{R1} = i_{C1} + i_{C2} + i_{R3}$

⑤ $i_{R2} = i_{R3} + i_{C2}$

⑥ $i_{C1} = C\frac{dU_{C1}}{dt}$

⑦ $i_{C2} = C\frac{dU_{C2}}{dt}$

⑧ $i_{R3} = C\frac{dU_A}{dt}$

⑧ in ③ $\rightarrow U_{C2} = RC\frac{dU_A}{dt} + U_A$ ③'

(⑦+⑧ in ⑤) + ③ in ② $\rightarrow$

$\rightarrow U_{C1} = R\cdot\left(C\frac{dU_A}{dt} + C\frac{d}{dt}U_{C2}\right) + U_{C2} \rightarrow U_{C1} = \left(RC\frac{dU_A}{dt} + \left(R^2C^2\frac{d^2U_A}{dt^2} + (RC)\frac{dU_A}{dt}\right)\right) + (RC)\frac{dU_A}{dt} + U_A$

②' $U_{C1} = U_A + 3RC\frac{dU_A}{dt} + (RC)^2\frac{d^2U_A}{dt^2}$ (vgl voriges Bsp 'Light'!)

(⑥+⑦+⑧ in ④) mit ②', ③'

$i_{R1} = i_{C1} + i_{C2} + i_{R3} = C\left(\frac{dU_A}{dt} + \frac{dU_{C1}}{dt} + \frac{dU_{C2}}{dt}\right)$

$= C\left(\left(\frac{dU_A}{dt}\right) + \left(RC\frac{d^2U_A}{dt^2} + \frac{dU_A}{dt}\right) + \left(\frac{dU_A}{dt} + 3RC\frac{d^2U_A}{dt^2} + (RC)^2\frac{d^3U_A}{dt^2}\right)\right)$

$U_E = RC\left[3\frac{dU_A}{dt} + 4RC\frac{d^2U_A}{dt^2} + (RC)^2\frac{d^3U_A}{dt^3}\right] + U_A + 3RC\frac{dU_A}{dt} + (RC)^2\frac{dU_A}{dt^2}$

$U_E = U_A + 6RC\frac{dU_A}{dt} + 5(RC)^2\frac{d^2U_A}{dt^2} + (RC)^3\frac{d^3U_A}{dt^3}$

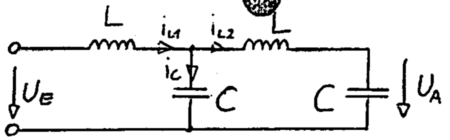
$u = y + 6\frac{dy}{d\tau} + 5\cdot\frac{d^2y}{d\tau^2} + \frac{d^3y}{d\tau^2} \quad / \mathcal{L}$

$U(s) = Y(s) + 6sY(s) + 5\cdot s^2Y(s) + s^3Y(s)$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^3 + 5s^2 + 6s + 1}$$

zg: Schaltung

s: $G(s)$ mit $T_B = \sqrt{LC}$



$$U_E = L\frac{di_{L1}}{dt} + U_C$$

$$U_C = L\frac{di_{L2}}{dt} + U_A = LC\frac{d^2U_A}{dt^2} + U_A$$

$$i_{L1} = i_C + i_{L2}$$

$$i_C = C\frac{dU_C}{dt}$$

$$i_{L2} = C\frac{dU_A}{dt}$$

in ③: $i_{L1} = C\frac{dU_C}{dt} + C\frac{dU_A}{dt} = C\frac{d}{dt}\left(L\frac{di_{L2}}{dt} + U_A\right) + C\frac{dU_A}{dt}$ mit ②

$i_{L1} = C\left[LC\frac{d^3U_A}{dt^2} + 2\frac{dU_A}{dt}\right]$ mit ⑤

$$U_E = L\frac{di_{L1}}{dt} + U_C = LC\left[LC\frac{d^4U_A}{dt^4} + 2\frac{d^2U_A}{dt^2}\right] + LC\frac{d^2U_A}{dt^2} + U_A$$

$$U_E = (LC)^2\frac{d^4U_A}{dt^4} + 3(LC)\frac{d^2U_A}{dt^2} + U_A$$

$y = \frac{U_A}{U_B}$ $u = \frac{U_E}{U_B}$

$\tau = \frac{t}{\sqrt{LC}}$

$$u \cdot U_B = \frac{(LC)^2}{(LC)^2}\cdot U_B \cdot \frac{d^4y}{d\tau^4} + 3\frac{LC}{LC}\cdot U_B\frac{d^2y}{d\tau^2} + y \cdot U_B$$

$dt = \sqrt{LC}\,d\tau \Rightarrow dt^2 = LC\,d\tau^2$

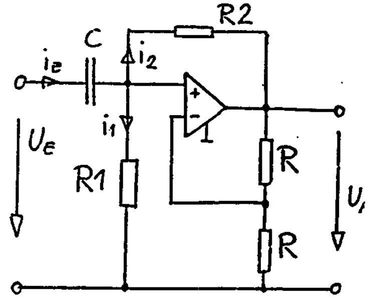
$$u = \frac{d^4y}{d\tau^4} + 3\frac{d^2y}{d\tau^2} + y \quad / \mathcal{L}$$

$dt^4 = (LC)^2 d\tau^4$

$$U(s) = s^4Y(s) + 3s^2Y(s) + Y(s)$$

$$\underline{\underline{G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^4 + 3s^2 + 1}}}$$

1.5) Geben Sie für die Schaltung mit einem idealen OPV die Differentialgleichung für die Ausgangsspannung U_A bei allgemeinen Verlauf der Eingangsspannung U_E an.



① $U_E = U_C + i_1 \cdot R_1$

② $i_E = i_1 + i_2$

③ $i_E = C\frac{dU_C}{dt}$ $\rightarrow C\frac{dU_C}{dt} = i_1 + i_2$

④ $i_1 \cdot R_1 = \frac{U_A}{2} \rightarrow i_1 = \frac{U_A}{2R_1}$

⑤ $i_1 \cdot R_1 = i_2 \cdot R_2 + U_A = \frac{U_A}{2} \longrightarrow i_2 = -\frac{U_A}{2R_2}$

① in ③: $C\frac{d}{dt}(U_E - i_1 \cdot R_1) = i_1 + i_2$

mit ④,⑤: $C\frac{d}{dt}\left(U_E - \frac{U_A}{2R_1}\cdot R_1\right) = \frac{U_A}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$C\frac{dU_E}{dt} = \frac{U_A}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{C}{2}\frac{dU_A}{dt}$$

$$\underline{\frac{dU_A}{dt} + \frac{1}{C}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\cdot U_A = 2\frac{dU_E}{dt}}$$

$u = \frac{U_E}{U_B}$; $y = \frac{U_A}{U_B}$

$T_B = C \cdot R_1$

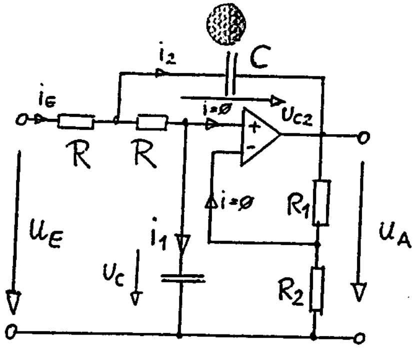
$\tau = \frac{t}{R_1C}$

$$\frac{U_B}{R_1C}\frac{dy}{d\tau} + \frac{1}{C}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\cdot U_B \cdot y = 2U_B\frac{1}{R_1C}\frac{du}{dt}$$

$$\underline{\underline{\frac{dy}{d\tau} + \left(1 - \frac{R_1}{R_2}\right)\cdot y = 2\frac{du}{d\tau}}}$$

…eiben Sie die Differential=
…eichung in bezogener Form an.

…etzen Sie dazu ideales
…erhalten des OPV voraus.



: $U_E = i_E\cdot R + i_1\cdot R + U_C \longrightarrow U_E = R(2i_1+i_2) + U_A\frac{R_2}{R_1+R_2} \Rightarrow u_E = f(i_1,i_2), f(U_A)$

: $U_C = U_A\cdot\frac{R_2}{R_1+R_2}$

: $i_E = i_1+i_2$

$i_1 = C\frac{du_C}{dt} = \frac{R_2\cdot C}{R_1+R_2}\frac{du_A}{dt}$ mit ② $\Rightarrow i_1 = f(U_A)$

$i_2 = C\frac{du_{C2}}{dt}$

$U_{C2} + U_A = i_1\cdot R + U_C$

n⑤ $\rightarrow i_2 = C\frac{d}{dt}(i_1\cdot R + U_C - U_A) = C\frac{d}{dt}\Big(\overset{④}{\frac{R_2}{R_1+R_2}\cdot RC\cdot\frac{du_A}{dt}} + \overset{②}{U_A\frac{R_2}{R_1+R_2}} - U_A\Big)$

$= C\frac{d}{dt}\Big(\frac{R_2}{R_1+R_2}\cdot RC\cdot\frac{du_A}{dt} - U_A\frac{R_1}{R_1+R_2}\Big) \Rightarrow i_2 = f(U_A)$

$u_E = R(2i_1+i_2) + U_A\frac{R_2}{R_1+R_2} =$

$= R\cdot C\cdot\Big[2\frac{R_2}{R_1+R_2}\frac{du_A}{dt} + \frac{R_2}{R_1+R_2}\cdot RC\cdot\frac{d^2u_A}{dt^2} - \frac{R_1}{R_1+R_2}\frac{du_A}{dt}\Big] + U_A\frac{R_2}{R_1+R_2}$

$u_E = \frac{R_2}{R_1+R_2}\cdot(RC)^2\frac{d^2u_A}{dt^2} + (RC)\frac{du_A}{dt}\cdot\Big(\frac{2R_2-R_1}{R_1+R_2}\Big) + \frac{R_2}{R_1+R_2}\,u_A$

$y = \frac{U_A}{U_B}$

$u = \frac{U_E}{U_B}$

$\tau = \frac{t}{RC}$

$u\cdot U_B = \frac{R_2}{R_1+R_2}\cdot U_B\frac{d^2y}{d\tau^2} + \Big(\frac{2R_2-R_1}{R_1+R_2}\Big)\cdot U_B\frac{dy}{d\tau} + \frac{R_2\cdot U_B}{R_1+R_2}\cdot y \quad /\cdot\frac{(R_1+R_2)}{R_2 U_B}$

$$\underline{\underline{\frac{d^2y}{d\tau^2} + \Big(2-\frac{R_1}{R_2}\Big)\frac{dy}{d\tau} + y = \Big(1+\frac{R_1}{R_2}\Big)\cdot u}}$$

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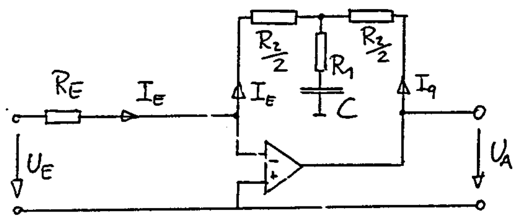
1.7) Geg: •) Inverentierender Verstärker mit Rückkoppelvierpol

OPV ideal



Ges: $G(s)$;

① $U_E = I_E\cdot R_E \Rightarrow I_E = \frac{U_E}{R_E}$

② $I_E\cdot\frac{R_2}{2} + (I_E+I_q)\cdot(R_1+\frac{1}{sC}) = \emptyset \Rightarrow \frac{U_E}{R_E}\cdot(\frac{R_2}{2}+R_1+\frac{1}{sC}) + I_q(R_1+\frac{1}{sC}) = \emptyset$

③ $U_A = I_q\cdot\frac{R_2}{2} - I_E\cdot\frac{R_2}{2} \Rightarrow U_A = I_q\cdot\frac{R_2}{2} - U_E\frac{R_2}{2R_E}$

$\rightarrow I_q$ eliminieren $I_q = -\frac{U_E}{R_1+\frac{1}{sC}}\cdot\Big(\frac{R_2}{2R_E}+\frac{R_1}{R_E}+\frac{1}{sR_EC}\Big)$

I_q in ③: $U_A = U_E\cdot\Bigg(-\frac{R_2}{2R_E} - \frac{\frac{R_2}{2}\Big(\frac{R_2}{2R_E}+\frac{R_1}{R_E}+\frac{1}{sR_EC}\Big)}{R_1+\frac{1}{sC}}\Bigg)$

$\underline{G(s)} = \frac{U_A}{U_E} = -\frac{R_2}{2R_E} - \frac{R_2/2}{R_1+1/sC}\Big(\frac{R_2}{2R_E}+\frac{R_1}{R_E}\Big) - \frac{R_2/sR_EC\cdot 2}{R_1+1/sC} = -\frac{R_2}{2R_E} - \frac{sC\frac{R_2}{2}}{1+sR_1C}\Big(\frac{R_2}{2R_E}+\frac{R_1}{R_E}\Big) - \frac{R_2/R_E 2}{1+sR_1C}$

$\underline{= -\frac{R_2}{2R_E} - \frac{sCR_1}{1+sCR_1}\cdot\Big(\frac{R_2^2}{2R_ER_1}+\frac{R_2}{2R_E}\Big) - \frac{1}{1+sCR_1}\cdot\frac{R_2}{2R_E}}$ $\qquad G(\emptyset) = -\frac{R_2}{R_E}$ ✓

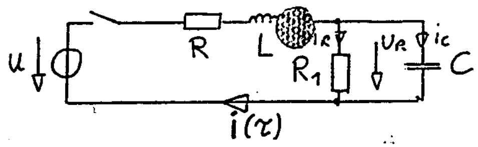
Normierung: $\tau = \frac{t}{T_B} = \frac{t}{R_1C}$; $dt = R_1C\,d\tau$; $y \circ\!\!-\!\!\bullet\, Y(s')$ $\quad \frac{dy(t)}{dt} = \frac{1}{R_1C}\frac{dy(\tau)}{d\tau} \circ\!\!-\!\!\bullet\, \frac{s'}{R_1C}Y(s')$

$\underline{\underline{G(s')}} = -\frac{R_2}{2R_E} - \frac{s'}{1+s'}\Big(\frac{R_2^2}{2R_ER_1}+\frac{R_2}{2R_E}\Big) - \frac{1}{1+s'}\frac{R_2}{2R_E} = \underline{\underline{\frac{-R_2}{2R_E}\cdot\Big(1+\frac{s'}{1+s'}\Big(\frac{R_2}{R_1}+1\Big)+\frac{1}{1+s'}\Big)}}$

eg: Schaltung

es: ? (starke Vermutung: DGL + Anfangsbed?)

i gegeben?



$= L\frac{di}{dt} + R\cdot i + u_R$ ①

$i_R + i_C$

$= \frac{u_R}{R_1}$

$= C\frac{du_R}{dt}$

$i = \frac{u_R}{R_1} + C\frac{du_R}{dt}$ ②

$\rightarrow u_R = u - L\frac{di}{dt} - Ri$

in ② $i = \frac{1}{R_1}\left(u - L\frac{di}{dt} - Ri\right) + C\frac{d}{dt}\left(u - L\frac{di}{dt} - Ri\right)$

$i = \frac{u}{R_1} - \frac{L}{R_1}\frac{di}{dt} - \frac{R}{R_1}\cdot i + C\frac{du}{dt} - LC\frac{d^2i}{dt^2} - RC\frac{di}{dt}$

$\Rightarrow LC\frac{d^2i}{dt^2} + \left(\frac{L}{R_1} - RC\right)\frac{di}{dt} + \frac{R}{R_1}\cdot i + i = C\frac{du}{dt} + \frac{u}{R_1}$

$\tau = \frac{t}{T_0} = \frac{t}{\sqrt{LC}}$

$y = \frac{i}{I_B}$

$x = \frac{u}{U_B} = \frac{u}{R\cdot I_B}$

$$\frac{d^2y}{d\tau^2} + \left(\sqrt{\frac{L}{C}}\cdot\frac{1}{R_1} - R\sqrt{\frac{C}{L}}\right)\frac{dy}{d\tau} + \left(\frac{R}{R_1}+1\right)y = R\sqrt{\frac{C}{L}}\frac{dx}{d\tau} + \frac{R}{R_1}x$$

$\frac{dy}{d\tau} \circ\!\!-\!\!\bullet\ sY(s) - y(0^-)$

$\frac{d^2y}{d\tau^2} \circ\!\!-\!\!\bullet\ s^2Y(s) - sy(0^-) - y^{(1)}(0^-)$

$$s^2Y(s) - sy(0^-) - y^{(1)}(0^-) + \left(\frac{\sqrt{L/C}}{R_1} - \frac{R}{\sqrt{L/C}}\right)\cdot\left(sY(s) - y(0^-)\right) + \left(\frac{R}{R_1}+1\right)Y(s) = \frac{R}{\sqrt{L/C}}\left(sX(s) - x(0^-)\right) + \frac{R}{R_1}X(s)$$

$$(s) = Y(s) = \underbrace{\frac{\left(R\sqrt{\frac{C}{L}}\,s + \frac{R}{R_1}\right)\cdot X(s)}{s^2 + \left(\frac{1}{R_1}\sqrt{\frac{L}{C}} - R\sqrt{\frac{C}{L}}\right)s + \left(\frac{R}{R_1}+1\right)}}_{Y_{0Z}(s)} + \underbrace{\frac{sy(0^-) + y^{(1)}(0^-) + y(0^-)\left(\frac{\sqrt{L/C}}{R_1} - \frac{R}{\sqrt{L/C}}\right) - \frac{R}{\sqrt{L/C}}x(0^-)}{s^2 + (\ \ )\,s + \left(\frac{R}{R_1}+1\right)}}_{Y_{0E}(s)}$$

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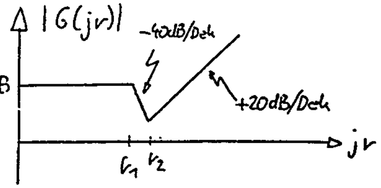
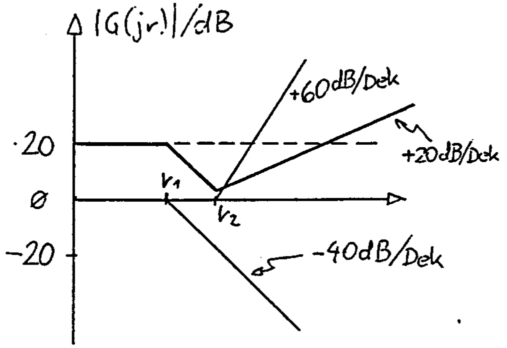
2.1) Geg: Minimalwinkelsystem



Ges: $G(j\nu)$



$$G(j\nu) = 10\,\frac{\left(\frac{j\nu}{\nu_2}+1\right)^3}{\left(\frac{j\nu}{\nu_1}+1\right)^2}$$

2.2) Geg: Bodediagramm von $G(s) = \frac{\left(\frac{s}{10}\right)^2 + 1}{10s + 10}$

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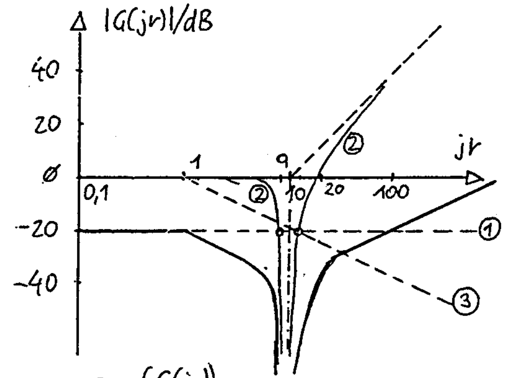
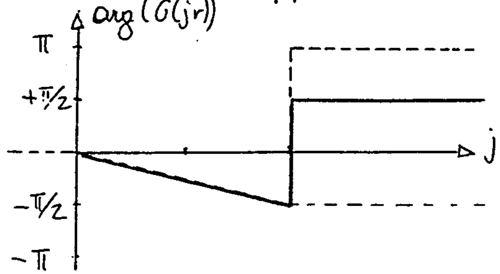


$G(j\nu) = \frac{1 - \left(\frac{\nu}{10}\right)^2}{10(j\nu + 1)}$

$\lg|G(j\nu)| = \lg\frac{1}{10} + \lg\left|1 - \left(\frac{\nu}{10}\right)^2\right| - \lg|j\nu + 1|$

Nullstelle bei $\nu = 10$! $\rightarrow |G(j10)| = -\infty$

Phasensprung!



$\arg(G(j\nu)) = \arg\left(1 - \left(\frac{\nu}{10}\right)^2\right) - \arg(j\nu + 1)$

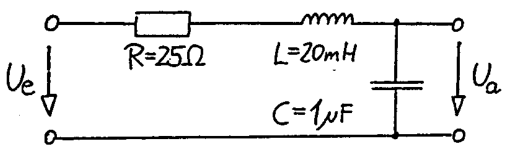
$\arg\left(1 - \left(\frac{\nu}{10}\right)^2\right) = 0 \quad \nu < 10$

$\pi \quad \nu > 10$

$\arg(1 + j\nu) = 0 \quad \nu \ll 1$

$\pi/2 \quad \nu \gg 1$

Geg: Berechnen und skizzieren Sie den Betragsgang des Bodediagramms der Übertragungsfunktion folgender Schaltung (normierte Frequenz $\nu = \omega\sqrt{LC}$)

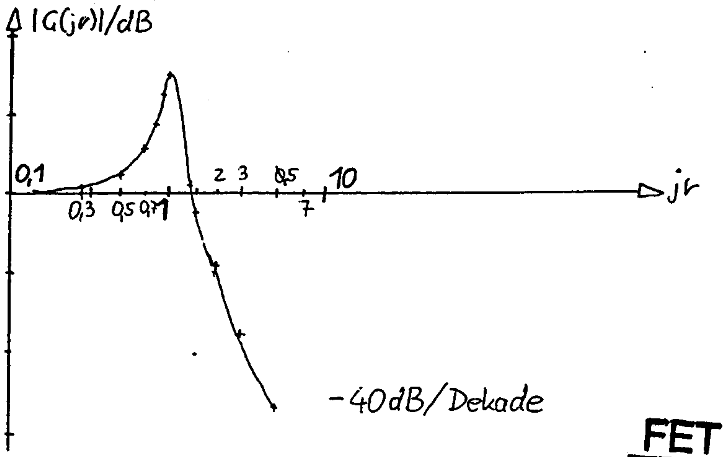


$$G(j\nu) = \frac{1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{1}{1-\omega^2 LC + j\omega RC}$$

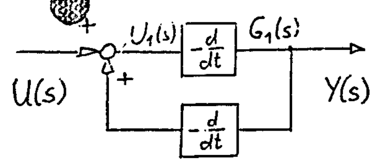
$$\nu = \omega\sqrt{LC} \Rightarrow \underline{G(j\nu)} = \frac{1}{1-\nu^2 + j\nu R\sqrt{C/L}} = \underline{\frac{1}{1-\nu^2 + j\,0{,}1768\nu}}$$

$$\underline{\underline{|G(j\nu)| = \frac{1}{\sqrt{(1-\nu^2)^2 + 0{,}1768^2\nu^2}}}}$$

$$20\log|G(j\nu)| = -10\log\left[(1-\nu^2)^2 + 0{,}1768^2\nu^2\right]$$



2.4) Geg:



Ges: Bodediagramm

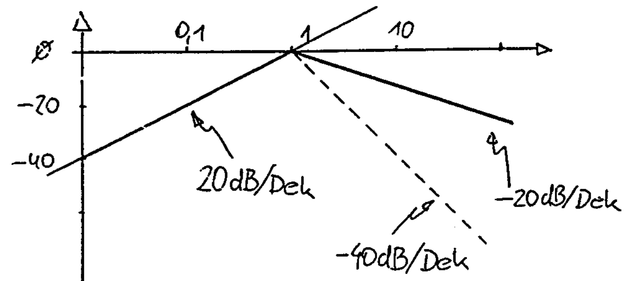
$$Y(s) = G_1(s)U_1(s) = -s\,U_1(s) \qquad G_1(s) = -s$$

$$Y(s) = -s(U(s) - sY(s)) \Rightarrow Y(s) = G(s)\cdot U(s) = \frac{-s}{-s^2+1}U(s)$$

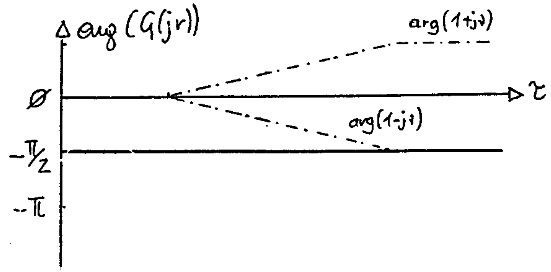
$$G(s) = \frac{-s}{-s^2+1} \rightarrow \underline{G(j\nu) = \frac{-j\nu}{\nu^2+1}}$$

$$\lg|G(j\nu)| = \lg|\nu| - \lg|1+\nu^2|$$

$$\arg(G(j\nu)) = \arg(-j\nu) - \arg(\nu^2+1) = -\arg(j\nu) = -\frac{\pi}{2}$$



Nenner: $1-s^2 = (1-s)(1+s)$
$\hat{=} (1-j\nu)(1+j\nu)$
$\arg((1-j\nu)(1+j\nu)) = \arg(1-j\nu) + \arg(1+j\nu)$
$= \emptyset$



Geg: Ein Lineares zeitinvariantes System wird durch die Differentialgleichung:

$$y^{(1)}+y = u^{(1)} - 0{,}1u$$

beschrieben. Zeichnen Sie den Betragsteil und den Winkelteil des Bode-Diagramms.

$$Y(s) = G(s)\cdot U(s)$$

$$sY(s) + Y(s) = U(s)\cdot s - 0{,}1\,U(s) \quad \Rightarrow \quad \underline{G(s)} = \frac{s-0{,}1}{s+1} = \underline{\frac{1}{10}\cdot\frac{s/0{,}1 - 1}{s+1}}$$

$$G(j\nu) = \frac{j\nu/0{,}1 - 1}{j\nu + 1}\cdot\frac{1}{10}$$

$$|G(j\nu)| = \frac{1}{10}\cdot\frac{|\frac{j\nu}{0{,}1}-1|}{|j\nu+1|} \Rightarrow \lg|G(j\nu)| = \overset{a}{\lg 0{,}1} + \overset{b}{\lg\left|\frac{j\nu}{0{,}1}-1\right|} - \overset{c}{\lg|j\nu+1|}$$

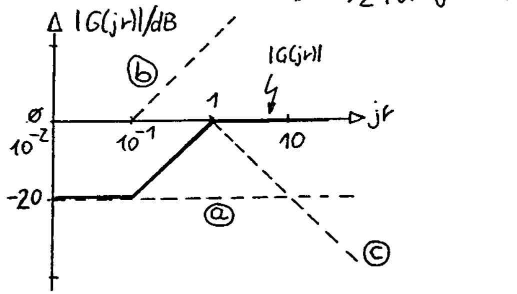
$$\arg(G(j\nu)) = \arg\left(\left(\frac{j\nu}{0{,}1}-1\right)\right) + \arg\left(\frac{1}{j\nu+1}\right)$$

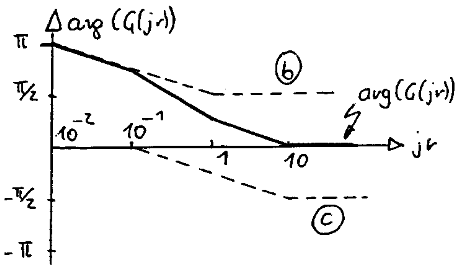
$$\arg\left(\frac{j\nu}{0{,}1}-1\right) \rightarrow \pi \text{ für } j\nu \ll 0{,}1$$

$$\rightarrow \pi/2 \text{ für } j\nu \gg 0{,}1$$

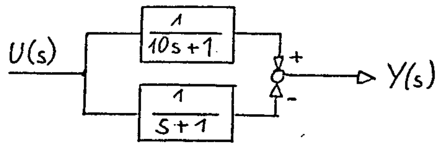
$$\arg\left(\frac{1}{j\nu+1}\right) \rightarrow -\emptyset \text{ für } j\nu \ll 1$$

$$\rightarrow -\pi/2 \text{ für } j\nu \gg 1$$





2.6) Geg:



Ges: Bodediagramm (Betrags-& Phase Frequenzdarstellung)

$$Y(s) = \left(\frac{1}{10s+1} - \frac{1}{s+1}\right)\cdot U(s) = G(s)\,U(s)$$

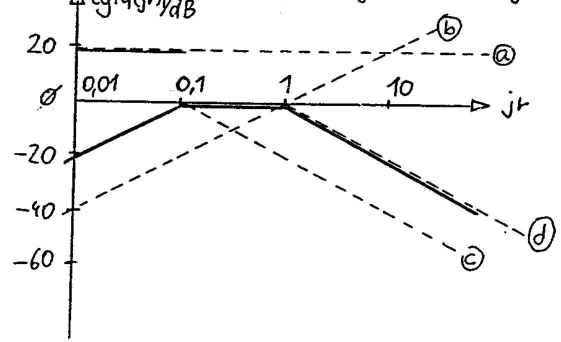
$$\Rightarrow G(s) = \frac{1}{10s+1} - \frac{1}{s+1} = \frac{s+1-10s-1}{(10s+1)(s+1)} = \frac{-9s}{(10s+1)(s+1)}$$

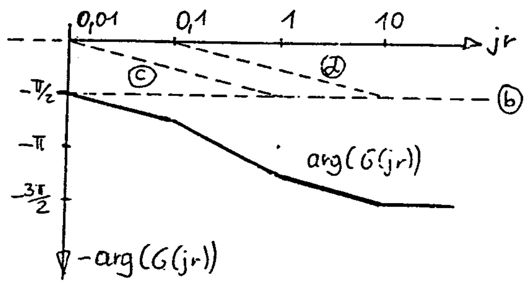
$$G(j\nu) = \frac{-j9\nu}{(\frac{j\nu}{0{,}1}+1)(j\nu+1)} = 9\cdot\frac{(-j\nu)}{(\frac{j\nu}{0{,}1}+1)(j\nu+1)}$$

$$\lg|G(j\nu)| = \overset{a}{\lg 9} + \overset{b}{\lg|\nu|} - \overset{c}{\lg\left|\frac{j\nu}{0{,}1}+1\right|} - \overset{d}{\lg|j\nu+1|}$$

$$\arg(G(j\nu)) = \arg(-j\nu) - \arg\left(\frac{j\nu}{0{,}1}+1\right) - \arg(j\nu+1)$$

$$= -\frac{\pi}{2} - \arg\left(\frac{j\nu}{0{,}1}+1\right) - \arg(j\nu+1)$$



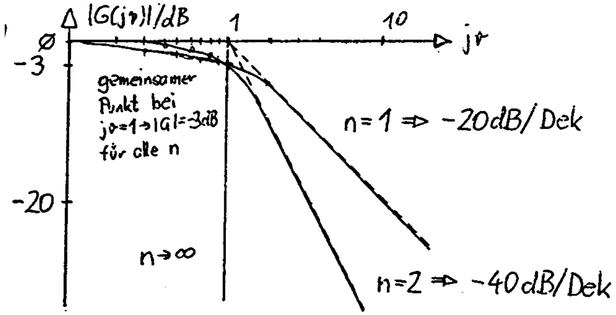


Geg: Ein BUTTERWORTH-Tiefpaßfilter n-ten Grades besitzt den Betragsfrequenzgang

$$|G(j\nu)| = \frac{1}{\sqrt{1+(\frac{\nu}{\nu_0})^{2n}}}$$

Ges: 1) Skizzieren Sie die Betragsteile der zugehörigen Bodediagramme für $n=2, n=1$ & $n\to\infty$.

2) Wie groß muß der Filtergrad n mindestens sein, daß das Dämpfungsmaß $-\ln|G(j\nu)|$ im Bereich $\nu \le 0{,}8\nu_0$ kleiner als 1dB ist? (n ist eine natürliche Zahl)



2) $|G(j\nu)| \hat{=} -1dB \Rightarrow |G(j\nu)| = 10^{-1/20} = 0{,}891$

$$|G(j\nu)| = \frac{1}{\sqrt{1+(\frac{\nu}{\nu_0})^{2n}}} = \frac{1}{\sqrt{1+0{,}8^{2n}}} > 0{,}891$$

$$\Rightarrow \left(\frac{1}{0{,}891}\right)^2 - 1 > 0{,}8^{2n} \Rightarrow n > \frac{1}{2}\,\frac{\lg\left[\left(\frac{1}{0{,}891}\right)^2-1\right]}{\lg 0{,}8} = 3{,}02$$

$\underline{\underline{n=3}} \Rightarrow |G(j0{,}8)| \hat{=} -1{,}01\,dB$

$n=4 \Rightarrow |G(j0{,}8)| \hat{=} -0{,}67\,dB$

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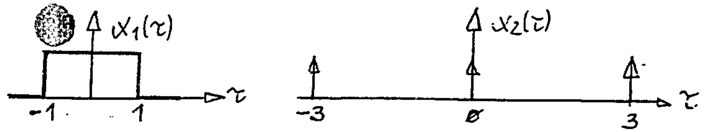
3.1) Geg:



Ges: $x_3(\tau) = x_1(\tau) * x_2(\tau)$

$x_1(\tau) = rect(\tau/2) \qquad x_2(\tau) = \delta(\tau+3)+\delta(\tau)+\delta(\tau-3)$

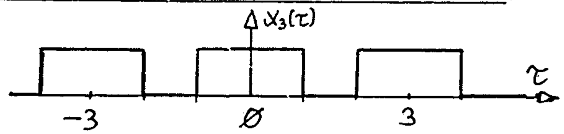
$$x_3(\tau) = x_1(\tau) * x_2(\tau) = \int_{-\infty}^{\infty} x_1(\tau')\,x_2(\tau-\tau')\,d\tau' = \int_{-\infty}^{\infty} x_1(\tau-\tau')\,x_2(\tau')\,d\tau'$$

$$x_3(\tau) = \int_{-\infty}^{\infty} rect\left(\frac{\tau'}{2}\right)\left(\delta(\tau-\tau'+3)+\delta(\tau-\tau')+\delta(\tau-\tau'-3)\right)d\tau'$$

$$= \int_{-\infty}^{\infty} rect\left(\frac{\tau'}{2}\right)\delta(\tau-\tau'+3)\,d\tau' + \int_{-\infty}^{\infty} rect\left(\frac{\tau'}{2}\right)\delta(\tau-\tau')\,d\tau' + \int_{-\infty}^{\infty} rect\left(\frac{\tau'}{2}\right)\delta(\tau-\tau'-3)\,d\tau'$$

Regel: $\int_{-\infty}^{\infty} x_1(\tau')\,\delta(\tau-\tau')\,d\tau' = x_1(\tau)$

$$\underline{\underline{x_3(\tau) = rect\left(\frac{\tau+3}{2}\right) + rect\left(\frac{\tau}{2}\right) + rect\left(\frac{\tau-3}{2}\right)}}$$



3.2) Geg: $x_1 = x(\tau) \qquad x_2 = x(\tau-\tau_0)$

Ges: $\varphi_{21}, \varphi_{12}$ als Funktion von φ_{11}

$$\varphi_{11} = x_1(\tau) \otimes x_1(\tau) = \int_{-\infty}^{\infty} x_1^*(\tau')\,x_1(\tau+\tau')\,d\tau' = \int_{-\infty}^{\infty} x^*(\tau')\cdot x(\tau+\tau')\,d\tau'$$

$$\underline{\underline{\varphi_{21}}} = x_2(\tau) \otimes x_1(\tau) = \int_{-\infty}^{\infty} x_2^*(\tau')\,x_1(\tau+\tau')\,d\tau' = \int_{-\infty}^{\infty} x_1^*(\tau'-\tau_0)\,x_1(\tau+\tau')\,d\tau' \qquad \tau'-\tau_0 = \tau''$$

$$= \int x_1^*(\tau'')\,x_1(\tau+\tau_0+\tau'')\,d\tau'' = \underline{\underline{\varphi_{11}(\tau+\tau_0)}}$$

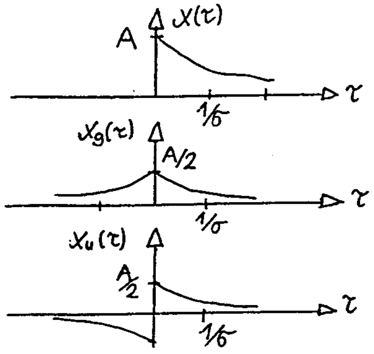
$$\underline{\underline{\varphi_{12}}} = x_1(\tau) \otimes x_2(\tau) = \int_{-\infty}^{\infty} x_1^*(\tau')\,x_2(\tau+\tau')\,d\tau' = \int_{-\infty}^{\infty} x^*(\tau')\,x(\tau+\tau'-\tau_0)\,d\tau'$$

$$= \int x^*(\tau)\,x(\tau-\tau_0+\tau')\,d\tau' = \underline{\underline{\varphi_{11}(\tau-\tau_0)}}$$

⇒ nicht kommutativ!

Geg: $x(\tau) = A e^{-\sigma\tau}\varepsilon(\tau)$

Ges: Berechnen & skizzieren Sie den geraden & ungeraden Anteil.



$$\underline{x_g(\tau)} = \frac{1}{2}\left[x(\tau)+x(-\tau)\right] = \underline{\underline{\frac{A}{2}\left[e^{-\sigma\tau}\varepsilon(\tau)+e^{\sigma\tau}\varepsilon(-\tau)\right]}}$$

$$\underline{x_u(\tau)} = \frac{1}{2}\left[x(\tau)-x(-\tau)\right] = \underline{\underline{\frac{A}{2}\left[e^{-\sigma\tau}\varepsilon(\tau)-e^{\sigma\tau}\varepsilon(-\tau)\right]}}$$

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3.4) Geg: $x(\tau) = \delta(\tau) + \delta(\tau-\tau_1)$

Ges: AKF $\varphi_{11}(\tau)$

3 verschiedene Methoden

a) $\varphi_{11}(\tau) = x(\tau) \otimes x(\tau) = \int_{-\infty}^{\infty} x^*(\tau')\, x(\tau+\tau')\, d\tau'$

für reelle Zeitfunktionen: $x^*(\tau) = x(\tau)$

$$\varphi_{11}(\tau) = \int_{-\infty}^{\infty} \left(\delta(\tau')+\delta(\tau'-\tau_1)\right)\cdot\left(\delta(\tau+\tau')+\delta(\tau+\tau'-\tau_1)\right) d\tau' = ① + ② + ③ + ④$$

$$① = \int_{-\infty}^{\infty} \delta(\tau')\cdot\delta(\tau+\tau')\, d\tau' = \delta(-\tau)$$

$$② = \int_{-\infty}^{\infty} \delta(\tau')\cdot\delta(\tau+\tau'-\tau_1)\, d\tau' = \delta(\tau_1-\tau)$$

$$③ = \int_{-\infty}^{\infty} \delta(\tau'-\tau_1)\cdot\delta(\tau+\tau')\, d\tau' = \int_{-\infty}^{\infty} \delta(\tau'')\cdot\delta(\tau+\tau''+\tau_1)\, d\tau'' = \delta(-\tau-\tau_1) \qquad \tau''=\tau'-\tau_1,\ \tau'=\tau''+\tau_1$$

$$④ = \int_{-\infty}^{\infty} \delta(\tau'-\tau_1)\,\delta(\tau+\tau'-\tau_1)\, d\tau' = \int_{-\infty}^{\infty} \delta(\tau'')\,\delta(\tau+\tau''+\tau_1-\tau_1)\, d\tau'' = \delta(-\tau)$$

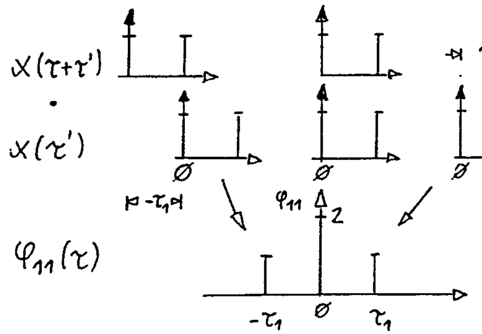
$$\underline{\varphi_{11}(\tau) = \delta(\tau_1-\tau) + 2\delta(-\tau) + \delta(-\tau-\tau_1)} \qquad / \ (-\tau)\to(\tau)$$

b) $x(\tau) = \delta(\tau) + \delta(\tau-\tau_1) \circ\!\!-\!\!\bullet\ X(j\nu) = 1 + e^{-j\nu\tau_1}$

$$\mathcal{F}\{\varphi_{11}(\tau)\} = |X(j\nu)|^2 = X(j\nu)\cdot X^*(j\nu) = (1+e^{-j\nu\tau_1})(1+e^{j\nu\tau_1}) = e^{-j\nu\tau_1} + 1 + e^{j\nu\tau_1}$$

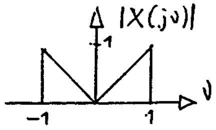
$$\underline{\underline{\varphi_{11}(\tau)}} = \mathcal{F}^{-1}\{e^{j\nu\tau_1} + 1 + e^{-j\nu\tau_1}\} = \underline{\underline{\delta(\tau+\tau_1)+\delta(\tau)+\delta(\tau-\tau_1)}}$$

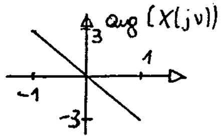
c) graphisch:



$$= \delta(\tau+\tau_1) + 2\delta(\tau) + \delta(\tau-\tau_1)$$

Geg:





Ges: $x(\tau)$

$$x(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\nu)e^{j\nu\tau}d\nu \qquad X(j\nu)=|\nu|e^{-j3\nu} \qquad |\nu|<1$$

$$x(\tau)=\frac{1}{2\pi}\left[\int_{-1}^{\emptyset}(-\nu)e^{-j3\nu}e^{j\tau\nu}d\nu+\int_{\emptyset}^{1}\nu e^{-j3\nu}e^{j\tau\nu}d\nu\right]$$

$$=\frac{1}{2\pi}\left[-\underbrace{\int_{-1}^{\emptyset}\nu e^{j\nu(\tau-3)}d\nu}_{1}+\underbrace{\int_{\emptyset}^{1}\nu e^{+j\nu(\tau-3)}d\nu}_{2}\right]$$

$$1:\ \sim=-\nu\frac{e^{j\nu(\tau-3)}}{j(\tau-3)}\Big|_{-1}^{\emptyset}+\frac{e^{j\nu(\tau-3)}}{(j(\tau-3))^2}\Big|_{-1}^{\emptyset} \qquad a=\nu\rightarrow da=d\nu$$

$$2:\ =+\nu\frac{e^{j\nu(\tau-3)}}{j(\tau-3)}\Big|_{\emptyset}^{1}-\frac{e^{j\nu(\tau-3)}}{(j(\tau-3))^2}\Big|_{\emptyset}^{1} \qquad db=e^{j\nu(\tau-3)}d\nu$$

$$1+2\quad =\frac{1}{j(\tau-3)}\left[-\emptyset-e^{-j(\tau-3)}+e^{j(\tau-3)}-\emptyset\right]+\frac{1}{(j(\tau-3))^2}\cdot\left[1-e^{-j(\tau-3)}-e^{j(\tau-3)}+1\right]$$

$$=\frac{2}{(\tau-3)}\sin(\tau-3)-\frac{2}{(\tau-3)^2}+\frac{2}{(\tau-3)^2}\cos(\tau-3)$$

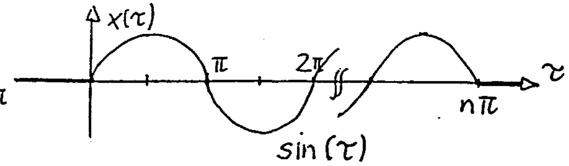
$$=2\,si(\tau-3)-si^2[(\tau-3)/2]$$

$$\Rightarrow\ \underline{\underline{x(\tau)=\frac{1}{2\pi}\left[2\,si(\tau-3)-si^2\left(\frac{\tau-3}{2}\right)\right]}}$$

$$X(j\nu)=\mathcal{F}\{x(\tau)\}=\left(rect(\nu/2)-rect(\nu)*rect(\nu)\right)\cdot e^{-j3\nu}$$

3.6) Geg: $x(\tau)=\emptyset \quad \tau<\emptyset$

$x(\tau)=\emptyset \quad \tau>n\pi$



Ges: $\mathcal{F}\{x(\tau)\}$

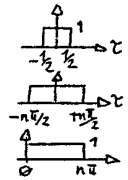
unterschiedliche Vorgangsweisen:

a) $x(\tau)=\sin(\tau)\varepsilon(\tau)-\sin(\tau)\varepsilon(\tau-n\pi)$

b) $x(\tau)=\sin(\tau)\cdot rect\left(\frac{\tau}{n\pi}-\frac{1}{2}\right)$

c) $x(\tau)=\int_{-\infty}^{\infty}x(\tau)e^{-j\nu\tau}d\tau=\int_{\emptyset}^{n\pi}\sin(\tau)e^{-j\nu\tau}d\tau$

b) gewählt



$x_1=rect(\tau)$ ○—● $X_1(j\nu)=si\left(\frac{\nu}{2}\right)$

$x_2=rect\left(\frac{\tau}{n\pi}\right)$ ○—● $X_2(j\nu)=n\pi\,si\left(\frac{n\pi\nu}{2}\right) \qquad n>\emptyset$

$x_3=rect\left(\frac{\tau-n\pi/2}{n\pi}\right)=rect\left(\frac{\tau}{n\pi}-\frac{1}{2}\right)$

○—● $X_3(j\nu)=n\pi\,si\left(\frac{n\pi\nu}{2}\right)\cdot e^{-j\left(\frac{n\pi}{2}\right)\nu}$

$$X(j\nu)=\mathcal{F}\{x(\tau)\}=\mathcal{F}\left\{\sin(\tau)\cdot rect\left(\frac{\tau}{n\pi}-\frac{1}{2}\right)\right\}=\mathcal{F}\{\sin(\tau)\}*\mathcal{F}\left\{rect\left(\frac{\tau}{n\pi}-\frac{1}{2}\right)\right\}$$

$$\mathcal{F}\{\sin(\tau)\}=\frac{\pi}{j}\left(\delta(\nu-1)-\delta(\nu+1)\right)$$

$$X(j\nu)=\frac{\pi}{j}\cdot n\cdot\pi\left[\left(\delta(\nu-1)-\delta(\nu+1)\right)*si\left(\frac{n\pi\nu}{2}\right)e^{-j\left(\frac{n\pi}{2}\right)\nu}\right]$$

$$=n\frac{\pi^2}{j}\left[si\left(\frac{n\pi}{2}(\nu-1)\right)e^{-j\left(\frac{n\pi}{2}\right)(\nu-1)}-si\left(\frac{n\pi}{2}(\nu+1)\right)\cdot e^{-j\left(\frac{n\pi}{2}\right)(\nu+1)}\right]$$

$$=n\frac{\pi^2}{j}\left[si\left(\frac{n\pi}{2}(\nu-1)\right)e^{+j\left(\frac{n\pi}{2}\right)}-si\left(\frac{n\pi}{2}(\nu+1)\right)e^{-j\left(\frac{n\pi}{2}\right)}\right]\cdot e^{-j\left(\frac{n\pi}{2}\right)\cdot\nu}$$

$$\underline{\underline{X(j\nu)=n\pi^2\left[si\left(\frac{n\pi}{2}(\nu-1)\right)e^{j\frac{\pi}{2}(n-1)}-si\left(\frac{n\pi}{2}(\nu+1)\right)e^{-j\frac{\pi}{2}(n+1)}\right]e^{-j\frac{\pi}{2}n\nu}}}$$

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Geg: •) System mit Stoßantwort $g(\tau)=\tau e^{-\tau}\varepsilon(\tau)$.

•) Eingangsfunktion $u(\tau)=\sum_k (-1)^k \delta(\tau-k\pi)$

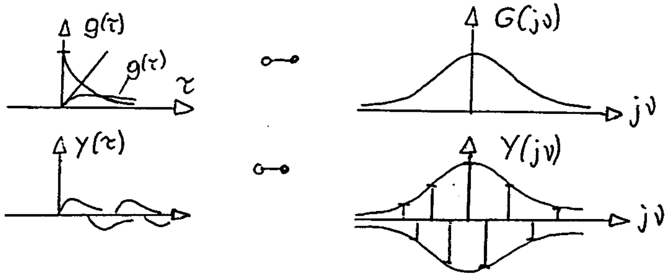
Ges: •) Spektrum am Ausgang $Y(j\nu)$

$$y(\tau)=g(\tau)*u(\tau)=\tau e^{-\tau}\varepsilon(\tau)*\sum_k (-1)^k \delta(\tau-k\pi)$$

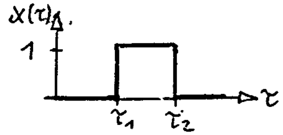
$$=\sum_k (-1)^k (\tau-k\pi) e^{-(\tau-k\pi)}\varepsilon(\tau-k\pi)$$

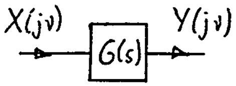
$$Y(j\nu)=\mathcal{F}\{y(\tau)\}=\sum_k (-1)^k \frac{e^{+jk\pi\nu}}{(j\nu+1)^2}$$

$$=\sum_k (-1)^k \frac{1}{(j\nu+1)^2}\cdot 2\pi\cdot\delta(\nu-2k)=\sum_k (-1)^k \frac{1}{(j(\nu-2k)+1)^2}\delta(\nu-2k)$$



3.8) Geg: Am Eingang des Filters mit der bekannten Übertragungsfkt. $G(s)$ liegt das angegebene Signal $x(\tau)$





Ges: Geben Sie die Spektralfunktion $Y(j\nu)$ des Ausgangssignals an.

$$x(\tau)=\int_{-\infty}^{\infty} X(j\nu)e^{j\nu\tau}d\nu\cdot\frac{1}{2\pi} \qquad X(j\nu)=\int_{-\infty}^{\infty} x(\tau)e^{-j\nu\tau}d\tau$$

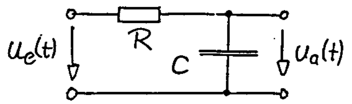
$$X(j\nu)=\int_{\tau_1}^{\tau_2}1\cdot 1\cdot e^{-j\nu\tau}d\tau=\left.\frac{e^{-j\nu\tau}}{-j\nu}\right|_{\tau_1}^{\tau_2}=\frac{e^{-j\nu\tau_2}-e^{-j\nu\tau_1}}{-j\nu}$$

$$Y(j\nu)=G(j\nu)\cdot X(j\nu)=\frac{G(j\nu)}{j\nu}\left(e^{-j\nu\tau_1}-e^{-j\nu\tau_2}\right)$$

$$G(j\nu)=G(s)\big|_{s=\sigma+j\nu} \text{ für } \sigma=0$$

Geg: •) Schaltung RC-Tiefpaß

•) Eingangsspg $u_e(t) = 12V + 4V\cos\omega t + 2V\cos(2\omega t + 30^\circ)$



$f_1 = 50Hz$

$R = 33{,}9\Omega \qquad C = 47\mu F$

Ges: •) $u_a(t)$ als Kosinusreihe

$$\frac{U_A}{U_E} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC} = G(j\omega)$$

$$u_e(t) = U_{0e} + U_{1e}\cos(\omega_1 t) + U_{2e}\cos(2\omega_1 t + 30^\circ)$$

$$= \mathrm{Re}\{U_{0e} + U_{1e}e^{j\omega_1 t} + U_{2e}e^{j2\omega_1 t}\cdot e^{j30^\circ}\}$$

$$u_a(t) = \mathrm{Re}\{U_{0a} + U_{1a}e^{j\omega_1 t} + U_{2a}e^{j2\omega_1 t}e^{j30^\circ}\} = G(j\omega)\cdot u_e(t)$$

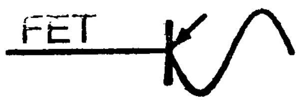
$$U_{0a} = G(0)\cdot U_{0e} = 1\cdot 12V = 12V$$

$$U_{1a} = G(j\omega_1)\cdot U_{1e} = \frac{1\cdot 4V}{1 + j2\pi 50\cdot 33{,}9\cdot 47\mu} = \frac{4V}{1{,}118\cdot e^{j26{,}6}} = 3{,}58\, e^{-j26{,}6^\circ}V$$

$$U_{2a} = G(j2\omega_1)\cdot U_{2e} = \frac{2V}{1 + j4\pi 50\cdot 33{,}9\cdot 47\mu} = \frac{2}{\sqrt{2}\cdot e^{j45^\circ}} = \sqrt{2}\, e^{-j45^\circ}V$$

$$u_a(t) = \mathrm{Re}\{12V + 3{,}58V e^{-j26{,}6^\circ}\cdot e^{j\omega_1 t} + \sqrt{2}V e^{j2\omega_1 t}e^{j30^\circ - 45^\circ}\}$$

$$\underline{\underline{u_a(t) = 12V + 3{,}58V\cos(\omega_1 t - 26{,}6^\circ) + 1{,}41V\cos(2\omega_1 t - 15^\circ)}}$$



4.2) Geg: $u(t) = \hat{U}\cdot\sin^3\omega t$

Ges: komplexe Fourier-Reihe

a) Rechenmethode

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$\sin^2 x = \left(\frac{e^{jx} - e^{-jx}}{2j}\right)^2 = \frac{e^{j2x} - 2 + e^{-j2x}}{-4}$$

$$\sin^3 x = \sin^2 x\cdot\sin x = \frac{e^{j2x} - 2 + e^{-j2x}}{-4}\cdot\frac{e^{jx} - e^{-jx}}{2j}$$

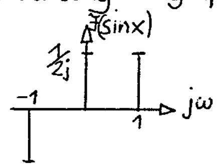
$$= \frac{1}{-8j}\left[e^{j3x} - 2e^{jx} + e^{-jx} - e^{jx} + 2e^{-jx} - e^{-j3x}\right]$$

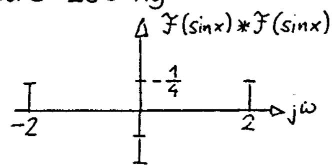
$$= \frac{1}{+8j}\left[-e^{j3x} + e^{-j3x} + 3e^{jx} - 3e^{-jx}\right] = \frac{1}{4}\left[3\sin x - \sin 3x\right]$$

$$u(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\omega kt} = \hat{U}\cdot\frac{1}{4}\left[3\sin\omega t - \sin 3\omega t\right]$$

$$= \frac{e^{-j3\omega t}}{8j} - \frac{3e^{-j\omega t}}{8j} + \frac{3e^{j\omega t}}{8j} - \frac{e^{j3\omega t}}{8j} \Rightarrow \quad c_{-3} = \frac{1}{8j} \quad c_{-1} = -\frac{3}{8j} \quad c_3 = -\frac{1}{8j} \quad c_1 = \frac{3}{8j}$$

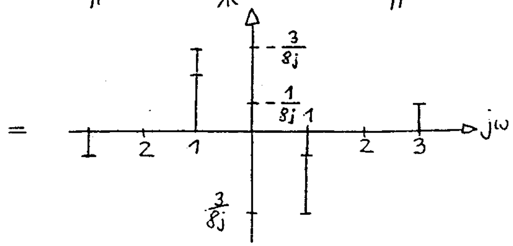
b) Faltung → graphische Lösung





$$\left(\frac{1}{2j}\right)^2 = -\frac{1}{4}$$

—||— $*$ —||— =



$$\left(\frac{1}{2j}\right)^3 = -\frac{1}{8j}$$

g: Die Fourier-Koeffizienten c_k eines 2π-periodischen Signals $x(\tau)$ seien bekannt. Leiten Sie die Beziehung ab zwischen den Koeffizienten c_{0k} des zeitverschobenen Signals $x(\tau-\tau_0)$ und den c_k.

$$c_k = \frac{1}{2\pi}\int_0^{2\pi} x(\tau)e^{-jk\tau}d\tau \qquad c_{0k} = \frac{1}{2\pi}\int_0^{2\pi} x(\tau-\tau_0)e^{-jk\tau}d\tau$$

$$c_{0k} = \frac{1}{2\pi}\int_{-\tau_0}^{2\pi-\tau_0} x(\tau')e^{-jk(\tau'+\tau_0)}d\tau' \qquad \tau'=\tau-\tau_0$$

$$= \frac{e^{-jk\tau_0}}{2\pi}\int_{-\tau_0}^{2\pi-\tau_0} x(\tau')e^{-jk\tau'}d\tau'$$

$$= \frac{e^{-jk\tau_0}}{2\pi}\left[\underbrace{\int_{-\tau_0}^{0} x(\tau')e^{-jk\tau'}d\tau'}_{1} + \int_{0}^{2\pi} x(\tau')e^{-jk\tau'}d\tau' + \underbrace{\int_{2\pi}^{2\pi-\tau_0} x(\tau')e^{-jk\tau'}d\tau'}_{3}\right]$$

$$1): \int_{-\tau_0}^{0} x(\tau')e^{-jk\tau'}d\tau' = \int_{2\pi-\tau_0}^{2\pi} x(\tau''-2\pi)e^{-jk\tau''}e^{-jk2\pi}d\tau'' \qquad \tau''=2\pi+\tau'$$

$$3): \int_{2\pi}^{2\pi-\tau_0} x(\tau')e^{-jk\tau'}d\tau' = -\int_{2\pi-\tau_0}^{2\pi} x(\tau')e^{-jk\tau'}d\tau' = -1$$

$$\Rightarrow \underline{\underline{c_{0k}}} = \frac{e^{-jk\tau_0}}{2\pi}\cdot\int_0^{2\pi} x(\tau')e^{-jk\tau'}d\tau' = \underline{\underline{e^{-jk\tau_0}c_k}}$$

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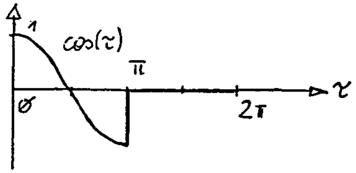
4.4) Geg: 2π-periodische Zeitfunktion $x(\tau)$

Ges: komplexe Amplitude der GS + 1.OS (Grundschwingung + 1. Oberschwingung)



$$c_k = \frac{1}{2\pi}\int_0^{2\pi} x(\tau)e^{-jk\tau}d\tau = \frac{1}{2\pi}\int_0^{\pi}\cos(\tau)e^{-jk\tau}d\tau$$

$$= \frac{1}{2\pi}\int_0^{\pi}\frac{e^{j\tau}+e^{-j\tau}}{2}e^{-jk\tau}d\tau = \frac{1}{4\pi}\left[\int_0^{\pi} e^{j(1-k)\tau}d\tau + \int_0^{\pi} e^{-j(1+k)\tau}d\tau\right]$$

$$= \frac{1}{4\pi}\left[\frac{e^{j(1-k)\tau}}{j(1-k)}\Big|_0^{\pi} + \frac{e^{-j(1+k)\tau}}{-j(1+k)}\Big|_0^{\pi}\right] = \frac{1}{4\pi}\left[\frac{e^{j\pi}\cdot e^{-jk\pi}-1}{j(1-k)} + \frac{e^{-j\pi}\cdot e^{-jk\pi}-1}{-j(1+k)}\right] \qquad e^{j\pi}=e^{-j\pi}\dots$$

$$= \frac{1}{4\pi}\left[(-e^{jk\pi}-1)\cdot\frac{-j-jk+j-jk}{1-k^2}\right] = \frac{jk}{2\pi}\cdot\frac{1}{k^2-1}\cdot(-(-1)^k-1)$$

$$= j\,\frac{1+(-1)^k}{2\pi}\cdot\frac{k}{k^2-1} \Rightarrow c_k = j\,\frac{1}{\pi}\,\frac{k}{k^2-1} \qquad |k|=3,5,7,\dots$$

$$k=1: \quad c_1 = \frac{1}{2\pi}\int_0^{\pi}\frac{1}{2}+\frac{e^{-j2\tau}}{2}d\tau = \frac{1}{2\pi}\left[\frac{\pi}{2}+\frac{e^{-j2\pi}-1}{-j4}\right] = \frac{1}{4}\left[1-j\frac{1}{2\pi}\right]$$

$$c_{-1} = \frac{1}{4}\left[1+j\frac{1}{2\pi}\right]$$

$\Rightarrow c_0 = 0$

$c_1 = \frac{1}{4}\left[1-\frac{jk}{2\pi}\right] = \frac{1}{4}\left[1-\frac{j}{2\pi}\right]$

$c_k = \frac{1}{4}\left[1-j\frac{k}{2\pi}\right] \quad k=-1,1$

$c_k = 0 \quad$ k gerade

$c_k = \frac{j}{\pi}\frac{k}{k^2-1} \quad$ k ungerade und $|k|>1$

$$\Rightarrow x(\tau) = \sum_{k=-\infty}^{\infty} c_k e^{jk\tau}$$

Grundschwingung: $|c_1| = \frac{1}{4}\sqrt{1+(\frac{1}{2\pi})^2}$

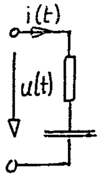
1. Oberschwingung: $k=3 \Rightarrow |c_3| = \frac{1}{\pi}\frac{3}{8} \qquad |c_{-3}| = \frac{1}{\pi}\cdot\frac{-3}{8}$

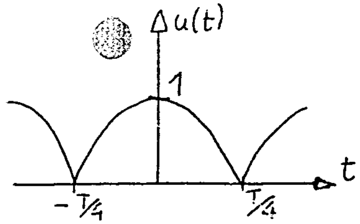
$$\underline{\underline{|x_0(\tau)|}} = |c_1|e^{jk\tau}+|c_1|e^{-jk\tau} = \left|\frac{1}{2}\sqrt{1+(\tfrac{1}{2\pi})^2}\cdot\cos k\tau\right| = \underline{\underline{\frac{\sqrt{1+(\frac{1}{2\pi})^2}}{2}}}$$

$$\underline{\underline{|x_1(\tau)|}} = |c_3|e^{j3k\tau}+|c_{-3}|e^{-j3k\tau} = \left|\frac{j3}{4\pi}\cdot\sin 3k\tau\right| = \underline{\underline{\frac{3}{4\pi}}}$$

eg: Zweiweggleichrichtung an R-C-Serienschaltung

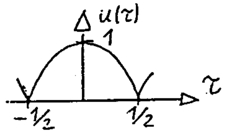
es: Fourierreihe des Stromes





$$c_{Uk} = \frac{1}{T_1}\int_0^{T_1} u(t)\, e^{-jk\omega_1 t}\,dt = \frac{1}{T/2}\int_{-T/4}^{T/4} \cos\left(\frac{2\pi}{T}\cdot t\right) e^{-jk\frac{2\pi}{T_1}\cdot t}\,dt$$

$$\tau = \frac{t}{T_1} = \frac{2\cdot t}{T} \rightarrow t = \frac{T}{2}\cdot\tau \;;\; dt = \frac{T}{2}\,d\tau$$



$$\Rightarrow c_{Uk} = \int_{-1/2}^{1/2} \cos(\pi\tau)\, e^{-jk2\pi\tau}\,d\tau$$

$\Rightarrow u(\tau)$ ist 1-periodisch! + gerade Funktion

$$= 2\int_0^{1/2} \frac{e^{j\pi\tau}+e^{-j\pi\tau}}{2}\, e^{-jk2\pi\tau}\,d\tau = \int_0^{1/2} e^{j(\pi-k2\pi)\tau}\,d\tau + \int_0^{1/2} e^{-j(\pi+k2\pi)}\,d\tau$$

$$= \frac{e^{j\frac{\pi-2\pi k}{2}}-1}{j(\pi-2\pi k)} + \frac{e^{-j\frac{\pi+2\pi k}{2}}-1}{-j(\pi+2\pi k)} = \frac{e^{j\frac{\pi}{2}}\cdot e^{-j\pi k}-1}{j(\pi-2\pi k)} + \frac{e^{-j\frac{\pi}{2}} e^{-j\pi k}-1}{-j(\pi+2\pi k)}$$

$e^{-j\pi k} = (-1)^k$

$e^{j\frac{\pi}{2}} = j$

$e^{-j\frac{\pi}{2}} = -j$

$$= \frac{1}{\pi^2-(2\pi k)^2}\cdot\Big(j(-1)^k\cdot(-j\pi-j2k\pi) + (j(\pi+2\pi k)) - j(-1)^k\cdot(j\pi-j2\pi k) - j(\pi-2\pi k)\Big)$$

$$= \frac{1}{\pi^2-(2\pi k)^2}\cdot\Big(j(-1)^k\cdot(-j\pi)\cdot 2 + j2\pi k\cdot 2\Big) = 2\cdot\frac{(-1)^k\pi + j2k\pi}{\pi^2-(2\pi k)^2} = \underline{\frac{2}{\pi}\cdot\frac{(-1)^k+j2k}{1-(2k)^2}}$$

$$G(j\omega) = \frac{U(j\omega)}{I(j\omega)} = R + \frac{1}{j\omega C} \qquad G(jk) = R + \frac{1}{jkC}$$

$$\rightarrow I(jk) = \frac{1}{R+1/jkC}\cdot U(jk) \rightarrow c_{Ik} = \frac{1}{R+1/jkC}\cdot c_{Uk}$$

$$\underline{c_{Ik}} = \frac{jkC}{1+jkRC}\cdot\frac{2}{\pi}\cdot\frac{(-1)^k+j2k}{1-(2k)^2} = \underline{\underline{\frac{Rk^2C^2+jkC}{1+(kRC)^2}\cdot\frac{2}{\pi}\cdot\frac{(-1)^k+j2k}{1-(2k)^2}}}$$

gültig für alle $k = 0, 1, 2, \ldots$; da einziger Pol bei $(2k)^2-1=0$ $\Rightarrow k = 1/2$ liegt.

4.8) Geg: • Lineares zeitinvariantes System mit der Stoßantwort

$$g(\tau) = \frac{\tau}{\tau_1}\, e^{-\frac{\tau}{\tau_1}}\,\varepsilon(\tau)$$

• Eingangsfunktion $u(\tau) = \sum_{n=-\infty}^{\infty} (-1)^n\,\delta(\tau-n\pi)$

Ges: Komplexe Fourier-Reihe der Ausgangsfunktion $y(\tau)$

$$y(\tau) = u(\tau) * g(\tau) = \sum_{n=-\infty}^{\infty} (-1)^n\,\delta(\tau-n\pi) * \frac{\tau}{\tau_1}\, e^{-\frac{\tau}{\tau_1}}\,\varepsilon(\tau)$$

$$= \sum_{n=-\infty}^{\infty} (-1)^n\,\frac{\tau-n\pi}{\tau_1}\, e^{-\frac{\tau-n\pi}{\tau_1}}\,\varepsilon(\tau-n\pi)$$

Periode $T=\pi$ $\Rightarrow$ $c_k = \frac{1}{T}\int_0^T y(\tau)\, e^{-jk\frac{2\pi}{T}\tau}\,d\tau = \frac{1}{\pi}\int_0^\pi y(\tau)\, e^{-jk2\tau}\,d\tau$

$$c_k = \frac{1}{\pi}\int_0^\pi \sum_{n=-\infty}^{\infty} (-1)^n\,\frac{\tau-n\pi}{\tau_1}\, e^{-\frac{\tau-n\pi}{\tau_1}}\,\varepsilon(\tau-n\pi)\, e^{-jk2\tau}\,d\tau$$

$$= \frac{1}{\pi}\underbrace{\sum_{n=-\infty}^{0} \varepsilon(\tau-n\pi)(-1)^n\, e^{\frac{n\pi}{\tau_1}}}_{1}\;\underbrace{\int_0^\pi \frac{\tau-n\pi}{\tau_1}\, e^{-\tau/\tau_1}\cdot e^{-jk2\tau}\,d\tau}_{2}$$

$$1 = \sum_{n=-\infty}^{0} (-1)^n\, e^{\frac{n\pi}{\tau_1}} = 1 - e^{-\frac{\pi}{\tau_1}} + e^{-\frac{2\pi}{\tau_1}} - \ldots = \frac{1}{1+e^{-\pi/\tau_1}}$$

genauer gerechnet bei einem $\mathcal{L}$-Bsp

$$2 = \int_0^\pi \frac{\tau-n\pi}{\tau_1}\, e^{-\tau(1/\tau_1+j2k)}\,d\tau = \frac{\tau-n\pi}{\tau_1}\,\frac{e^{-\tau(\,)}}{-1/\tau_1-j2k}\Bigg|_0^\pi - \frac{1}{\tau_1}\,\frac{e^{-\tau(\,)}}{(1/\tau_1+j2k)^2}\Bigg|_0^\pi$$

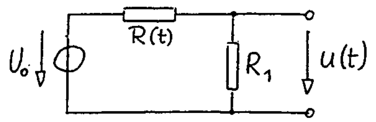
$$= \frac{1}{-1-j2k\tau_1}\cdot\left[\pi(1-n)\,e^{-\pi/\tau_1-j2\pi k} + n\pi\right] - \frac{1}{\tau_1(1/\tau_1+j2k)^2}\cdot\left[\left(e^{-\pi/\tau_1-j2\pi k}-1\right)\right]$$

$$= \frac{1}{1+j2k\tau_1}\cdot\left[n\pi\cdot\left(e^{-\pi/\tau_1}-1\right) - e^{-\pi/\tau_1}\pi\right] + \frac{\tau_1}{(1+j2k\tau_1)^2}\left[1-e^{-\pi/\tau_1}\right]$$

$$\underline{\underline{c_k}} = \frac{1}{\pi}\cdot 1\cdot 2 = \underline{\underline{\frac{1}{1+j2k\tau_1}\left[\frac{n\left(e^{-\pi/\tau_1}-1\right)-e^{-\pi/\tau_1}}{1+e^{-\pi/\tau_1}}\right] + \frac{\tau_1/\pi}{(1+j2k\tau_1)^2}\,\frac{1-e^{-\pi/\tau_1}}{1+e^{-\pi/\tau_1}}}}$$

FET

eg: ·)



$0<a<1$

·) zeitlich veränderlicher Widerstand $R(t)=R_0(1+a\cos(\omega t))$

es: Verhältnis der Amplituden der 1. Oberwelle zur Grundschwingung

---

$$\frac{t)}{_0} = \frac{R_1}{R_1+R_0(1+a\cos(\omega t))} = \frac{\frac{R_1}{R_0+R_1}}{1+a\frac{R_0}{R_0+R_1}\cos(\omega t)} = \frac{b}{1+c\cos(\omega t)}$$

$b=\frac{R_1}{R_0+R_1}$

$c=a\frac{R_0}{R_0+R_1}$

$$\frac{1}{1+x} = \sum_{k=0}^{\infty}(-x)^k \quad \text{für } |x|<1$$

$$\frac{t)}{_0} = b\cdot\sum_{k=0}^{\infty}(-c\cos(\omega t))^k = b-bc\cos(\omega t)+bc^2\cos^2(\omega t)-bc^3\cos^3(\omega t)+\dots$$

$$\cos x = \frac{e^{jx}+e^{-jx}}{2} \qquad \cos^2 x = \frac{(e^{jx}+e^{-jx})^2}{4} = \frac{1e^{j2x}+2+e^{-j2x}}{2\cdot 2} = \frac{1}{2}(1+\cos 2x)$$

$$\cos^3 x = \frac{e^{j3x}+3e^{jx}+3e^{-jx}+e^{-j3x}}{2^3} = \frac{1}{4}(\cos 3x+3\cos x)$$

$$\cos^4 x = \frac{e^{j4x}+4e^{j2x}+6+4e^{-j2x}+e^{-j4x}}{2^4} = \frac{1}{8}(\cos 4x+4\cos 2x+3)$$

$$\cos^5 x = \frac{e^{j5x}+5e^{j3x}+10e^{jx}+10e^{-jx}+\dots}{2^5} = \frac{1}{16}(\cos 5x+5\cos 3x+10\cos x)$$

1
1 1
$\boxed{1}$ 2 1
1 3 3 1
1 $\boxed{4}$ 6 4 1
1 5 10 10 5 1
1 6 $\boxed{15}$ 20

$$\frac{(t)}{_0} = \dots-bc\cos(\omega t)\cdot\left(1+\frac{3}{4}c^2+\frac{10}{16}c^4+\dots\right)+bc^2\cos^2(\omega t)\cdot\left(\frac{1}{2}+\frac{1}{2}c^2+\dots\right)$$

$$= \dots-bc\cos(\omega t)\cdot\sum_{k=0}^{\infty}\frac{c^{2k}}{2^{2k}}\cdot\binom{k}{2k-1}+bc\cos^2(\omega t)\cdot\sum_{k=1}^{\infty}\frac{c^{2k-1}}{2^{2k-1}}\binom{k}{2k}$$

$$\frac{_1}{_0} = \frac{bc\sum_{k=0}^{\infty}\frac{c^{2k}}{2^{2k}}\binom{k}{2k-1}}{bc\sum_{k=1}^{\infty}\frac{c^{2k-1}}{2^{2k-1}}\binom{k}{2k}} = \frac{\sum_{k=0}^{\infty}\left(\frac{c}{2}\right)^{2k}\binom{k}{2k-1}}{\sum_{k=1}^{\infty}\left(\frac{c}{2}\right)^{2k-1}\binom{k}{2k}}$$

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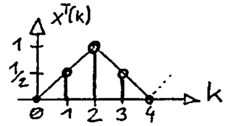
4.10) Geg: $\tilde{X}_d(jk)=\sum_{n=0}^{N-1}x_d(n)e^{-j2\pi n\frac{k}{N}}$ $\qquad x_d(n)=\frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}(jk)e^{j2\pi n\frac{k}{N}}$

Signal:



Ges: DFT

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$x^T(k)$: Periode $N=5$

Spektrum: $\tilde{X}_d(jk)=\sum_{n=0}^{4}x_d(n)e^{-j2\pi n\frac{k}{5}}$

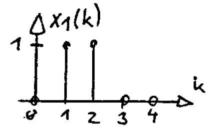
$$\tilde{X}_d(jk)=x_d(0)\cdot e^{-j0}+x_d(1)e^{-j2\pi\frac{k}{5}}+x_d(2)e^{-j4\pi\frac{k}{5}}+x_d(3)e^{-j6\pi\frac{k}{5}}+x_d(4)e^{-j8\pi\frac{k}{5}}$$

$$=0+\frac{1}{2}e^{-j2\pi\frac{k}{5}}+1e^{-j4\pi\frac{k}{5}}+\frac{1}{2}e^{-j2\pi\frac{k}{5}\cdot 3}+0$$

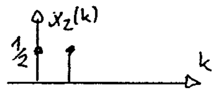
$$=e^{-j4\pi\frac{k}{5}}\cdot\left[\frac{1}{2}e^{j2\pi\frac{k}{5}}+1+\frac{1}{2}e^{-j2\pi\frac{k}{5}}\right]=e^{-j4\pi\frac{k}{5}}\cdot\left[1+\cos\left(2\pi\frac{k}{5}\right)\right]$$

$$=2e^{-j4\pi\frac{k}{5}}\cdot\left[\cos^2\left(\pi\frac{k}{5}\right)\right]=2\cdot\left[\cos\left(\pi\frac{k}{5}\right)\cdot e^{-j2\pi\frac{k}{5}}\right]^2$$

oder:



$\circ\!\!-\!\!\bullet$ $\tilde{X}_1(jk)=e^{-j3\pi\frac{k}{5}}\cos\left(k\frac{\pi}{5}\right)\cdot 2$



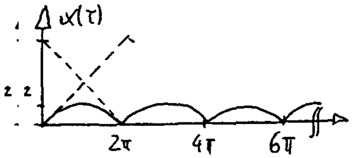
$\circ\!\!-\!\!\bullet$ $X_2(jk)=e^{-j\pi\frac{k}{5}}\cos\left(k\frac{\pi}{5}\right)$

$$x_d(k)=x_1(k)*x_2(k)=\int x_1(k')x_2^*(k-k')dk'=\sum_{\ell=0}^{4}x_1(\ell)x_2^*(k-\ell)$$

$$\tilde{X}_d(k)=\tilde{X}_1(jk)\cdot\tilde{X}_2(jk)=2\cdot e^{-j3\pi\frac{k}{5}}\cdot e^{-j\pi\frac{k}{5}}\cdot\cos^2\left(k\frac{\pi}{5}\right)=2e^{-j4\pi\frac{k}{5}}\cos^2\left(k\frac{\pi}{5}\right)$$

...g: 2π-periodisches Signal $x(\tau)=\tau(2\pi-\tau)$ $\quad 0<\tau<2\pi$

...s: Fourier-Reihe in reeller Darstellung



$$c_k=\frac{1}{2\pi}\int_0^{2\pi}x(\tau)e^{-jk\tau}d\tau=\frac{1}{2\pi}\int_0^{2\pi}x(\tau)\cdot\cos k\tau\,d\tau=\frac{1}{2\pi}\int_0^{2\pi}\tau(2\pi-\tau)\cos k\tau\,d\tau$$

$$=\frac{1}{2\pi}\Big[2\pi\underbrace{\int_0^{2\pi}\tau\cos k\tau\,d\tau}_{1}-\underbrace{\int_0^{2\pi}\tau^2\cos k\tau\,d\tau}_{2}\Big]$$

$$1=2\pi\Big[\tau\frac{\sin k\tau}{k}\Big|_0^{2\pi}+\frac{\cos k\tau}{k^2}\Big|_0^{2\pi}\Big]=2\pi[0-0+1-1]=0 \qquad a=\tau\ ;\ db=\cos k\tau\,d\tau$$

$$2=-\Big[\tau^2\cdot\frac{\sin k\tau}{k}\Big|_0^{2\pi}-2\int\tau\frac{\sin k\tau}{k}d\tau\Big] \qquad a=\tau^2 \quad db=\cos k\tau\,d\tau \quad da=2\tau d\tau \quad b=\frac{\sin k\tau}{k}$$

$$=-\Big[(2\pi)^2\cdot 0-\frac{2}{k}\Big[-\tau\frac{\cos k\tau}{k}\Big|_0^{2\pi}+\frac{\sin k\tau}{k}\Big|_0^{2\pi}\Big]\Big] \qquad a=\tau \quad db=\sin k\tau\,d\tau \quad b=-\frac{\cos k\tau}{k}$$

$$=\frac{2}{k}\Big[-\frac{2\pi}{k}+0+0-0\Big]=-2\pi\,\frac{2}{k}$$

$$c_k=\frac{1}{2\pi}\big[1+2\big]=-\frac{2}{k} \qquad k>0$$

Beachte: $1=0$ ⇒ Der Anteil $2\pi\cdot\tau$ von $x(\tau)$ fällt total heraus!

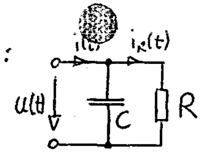
$$x(\tau)=2\pi\tau-\tau^2=-\tau^2+2\pi\tau-\pi^2+\pi^2=-(\tau-\pi)^2+\pi^2 \qquad \tau'=\tau-\pi$$

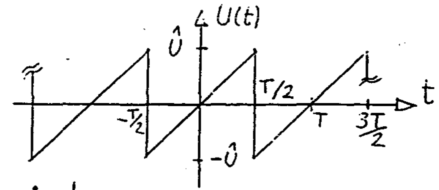
$$x(\tau')=-\tau'^2+\pi^2 \qquad -\pi<\tau'<\pi$$

$$c_0=\frac{1}{2\pi}\int_{-\pi}^{\pi}(-\tau)^2+\pi^2\,d\tau=\frac{1}{2\pi}\Big[\frac{+(-\tau)^3}{3}+\pi^2\tau\Big|_{-\pi}^{\pi}\Big]=\frac{1}{2\pi}\Big[\frac{\pi^3+\pi^3}{3}+\pi^3+\pi^3\Big]=\frac{4}{6}\pi^2$$

$$=\frac{1}{2\pi}\int_0^{2\pi}2\pi\tau-\tau^2\,d\tau=\frac{1}{2\pi}\Big[2\pi\frac{\tau^2}{2}\Big|_0^{2\pi}-\frac{\tau^3}{3}\Big|_0^{2\pi}\Big]=\frac{1}{2\pi}\Big[\frac{(2\pi)^3}{2}-\frac{(2\pi)^3}{3}\Big]=\frac{(2\pi)^2}{6}\ ✓$$

4.6) Geg:





Ges: Fourier-Reihe von i_R !

$G(jk)$:

$$u(t)=R\cdot i_R(t)\Rightarrow G(j\omega)=\frac{I(j\omega)}{U(j\omega)}=\frac{1}{R}=G(jk)$$

$u(t)$: $$c_{Uk}=\frac{1}{T}\int_{-T/2}^{T/2}u(t)e^{-jk\frac{2\pi}{T}t}dt \qquad \tau=\frac{t}{T} \quad u(t)=\frac{\hat{U}}{T/2}\cdot t=2\hat{U}\cdot\tau$$

⇒ $u(\tau)$ ist 1-periodisch

$$c_{Uk}=2\hat{U}\int_{-1/2}^{1/2}\tau e^{-jk2\pi\tau}d\tau \qquad a=\tau \quad db=e^{-jk2\pi\tau}d\tau$$

$$=2\hat{U}\Big[\tau\frac{e^{-jk2\pi\tau}}{-jk2\pi}\Big|_{-1/2}^{1/2}-\frac{e^{-jk2\pi\tau}}{(-jk2\pi)^2}\Big|_{-1/2}^{1/2}\Big] \qquad \int a\,db=a\cdot b-\int b\,da$$

$$=2\hat{U}\Big[\frac{1}{-jk2\pi}\Big(\frac{e^{-jk\pi}}{2}+\frac{e^{+jk\pi}}{2}\Big)-\frac{1}{-jk2\pi}\underbrace{\big(e^{-jk\pi}-e^{jk\pi}\big)}_{0}\Big] \qquad e^{-jk\pi}=e^{jk\pi}=(-1)^k$$

$$c_{Uk}=j\frac{\hat{U}}{k\pi}\cdot(-1)^k \qquad \text{für } k\neq 0$$

oder, weil $u(\tau)$ ungerade Funktion ist: $\qquad \sin x=-\sin -x$

$$c_{Uk}=\frac{1}{T}\int_{-T/2}^{+T/2}u(t)e^{-jk\frac{2\pi}{T}t}dt=\frac{2\hat{U}}{T^2}\int_{-T/2}^{T/2}t\underbrace{\big(\cos(k\tfrac{2\pi}{T}t)}_{\int\ldots dt=0}-j\sin(k\tfrac{2\pi}{T}t)\big)dt$$

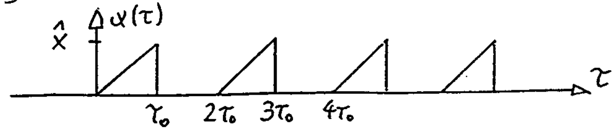
$$=-j\frac{2\hat{U}}{T^2}\int_{-T/2}^{T/2}t\sin(k\tfrac{2\pi}{T}t)\,dt=-j2\frac{2\hat{U}}{T^2}\int_0^{T/2}t\sin(k\tfrac{2\pi}{T}t)dt$$

$$c_{Uk}=-j2\hat{U}\int_0^{1/2}2\tau\sin(2\pi k\cdot\tau)d\tau=-j4\hat{U}\cdot\Big[\frac{-\tau\cos(2\pi k\tau)}{2\pi k}\Big|_0^{1/2}+\frac{\sin(2\pi k\tau)}{(2\pi k)^2}\Big|_0^{1/2}\Big]$$

$$=-j4\hat{U}\cdot\frac{-\frac{1}{2}\cdot\cos(k\pi)}{2\pi k}=j\frac{\hat{U}}{k\pi}\cdot(-1)^k \qquad |k|>0$$

$$c_{ik}=G(jk)\cdot c_{Uk}=j\frac{\hat{U}}{Rk\pi}\cdot(-1)^k$$

g: periodisches Signal $x(\tau)$



s: $X(s)$

$$\tau) = \sum_{k=0}^{\infty} \delta(\tau - k(2\tau_0)) * x_1(\tau) = \sum_{k=0}^{\infty} \tilde{x}_1(\tau - k(2\tau_0))$$

$$\tilde{x}_1(\tau) = \left[\tau\varepsilon(\tau) - \tau\varepsilon(\tau-\tau_0)\right] = \tau\varepsilon(\tau) - (\tau-\tau_0)\varepsilon(\tau-\tau_0) - \tau_0\varepsilon(\tau-\tau_0)\Big] \cdot \frac{\hat{x}}{\tau_0}$$

$$\tilde{X}_1(s) = \mathcal{L}(x_1(\tau)) = \frac{\hat{x}}{\tau_0} \cdot \left[\frac{1}{s^2} - \frac{e^{-\tau_0 s}}{s^2} - \tau_0 \frac{e^{-\tau_0 s}}{s}\right]$$

$$s) = \mathcal{L}(x(\tau)) = \mathcal{L}\left(\sum_{k=0}^{\infty} \delta(\tau - k(2\tau_0))\right) \cdot \mathcal{L}(\tilde{x}_1(\tau))$$

$$= \sum_{k=0}^{\infty} \mathcal{L}\{\delta(\tau - k(2\tau_0))\} \cdot \tilde{X}_1(s)$$

$$\sum_{k=0}^{\infty} \mathcal{L}\{\delta(\tau - k(2\tau_0))\} = \sum_{k=0}^{\infty} e^{-k2\tau_0 s} = \lim_{N\to\infty} \sum_{k=0}^{N} e^{-k2\tau_0 s} = \lim_{N\to\infty} \frac{1 - e^{-2\tau_0 s(N+1)}}{1 - e^{-2\tau_0 s}}$$

$$= \frac{1}{1 - e^{-2\tau_0 s}}$$

$$s) = \frac{1}{1 - e^{-2\tau_0 s}} \cdot \tilde{X}_1(s) = \frac{1}{1 - e^{-2\tau_0 s}} \cdot \frac{\hat{x}}{\tau_0}\left[\frac{1}{s^2}(1 - e^{-\tau_0 s}) - \frac{1}{s}\tau_0 e^{-\tau_0 s}\right]$$

$$= \frac{\hat{x}}{\tau_0}\left[\frac{1}{s^2}\frac{1 - e^{-\tau_0 s}}{1 - e^{-2\tau_0 s}} - \frac{1}{s}\frac{\tau_0 e^{-\tau_0 s}}{1 - e^{-2\tau_0 s}}\right]$$

$$= \frac{\hat{x}}{\tau_0}\left[\frac{1}{s^2}\frac{1}{1 + e^{-\tau_0 s}} - \frac{\tau_0}{s}\frac{e^{-\tau_0 s}}{1 - e^{-2\tau_0 s}}\right]$$

gl. mit Formel: $\mathcal{L}(x(\tau)) = \frac{1}{1 - e^{-s\tau_1}} \int_0^{\tau_1} x(\tau) e^{-s\tau} d\tau \qquad x(\tau) = x(\tau - \tau_1)$

FET

66) Geg: Angenommen, ein rechtsseitiges Signal läßt sich durch eine Potenzreihe gemäß

$$x(\tau) = \left[\sum_{n=-\infty}^{\infty} \frac{c_n}{n!} \tau^n\right] \varepsilon(\tau)$$

darstellen. Geben Sie eine entsprechende Reihenentwicklung für die Laplace-Transformierte $X(s)$ an

---

$$x(\tau) = \underbrace{\sum_{n=-\infty}^{-1} \frac{c_n}{n!} \cdot \tau^n \cdot \varepsilon(\tau)}_{1} + \underbrace{c_0 \cdot \varepsilon(\tau)}_{2} + \underbrace{\sum_{n=1}^{\infty} \frac{c_n}{n!} \tau^n \varepsilon(\tau)}_{3}$$

$= \sum \frac{c_{-n}}{(-n)!} \cdot \tau^{-n} \varepsilon(\tau) \circ\!\!-\!\!\bullet\ X_1(s) \cdot \sum \frac{c_{-n}}{-n!} \mathcal{L}(\tau^{-n}\varepsilon(\tau))$

?

1: $x_1(\tau) = \sum_{n=1}^{\infty} \frac{c_{-n}}{(-n)!} \cdot \frac{\varepsilon(\tau)}{\tau^n} \circ\!\!-\!\!\bullet\ X_1(s) = \sum_{n=1}^{\infty} \frac{c_{-n}}{(-n)!} \mathcal{L}\left(\frac{\varepsilon(\tau)}{\tau^n}\right)$

allgemeine Regel: $X(s) = \int_{0-}^{\infty} x(\tau) e^{-s\tau} d\tau \quad / \int ds^n$

$$\int X(s) ds^n = \int_{0-}^{\infty} \frac{x(\tau)}{\tau^n} e^{-s\tau} d\tau = \mathcal{L}\left(\frac{x(\tau)}{\tau^n}\right)$$

$\mathcal{L}\left(\frac{\varepsilon(\tau)}{\tau^n}\right) = \int \frac{1}{s} ds^n = \int \ln s \, ds^{n-1}$ für $n > 1$ $\qquad u(s) = \ln s;\ v'(s) = 1$

$$= \int\int \ln s \, ds \, ds^{n-2} = \int (s \ln s - s) ds^{n-2} = \ldots$$

$$\Rightarrow \underline{X_1(s) = \sum_{n=1}^{\infty} \frac{c_{-n}}{(-n)!} \mathcal{L}\left(\frac{\varepsilon(\tau)}{\tau^n}\right)}$$

2: $X_2(s) = \frac{c_0}{s}$

3: $x_3(\tau) = \sum_{n=1}^{\infty} \frac{c_n}{n!} \tau^n \varepsilon(\tau) \circ\!\!-\!\!\bullet\ X_3(s) = \sum_{n=1}^{\infty} \frac{c_n}{n!} \mathcal{L}(\tau^n \varepsilon(\tau))$

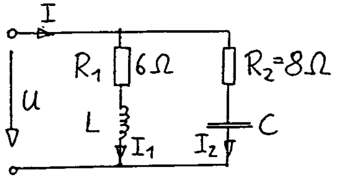
$$\mathcal{L}(\tau^n \varepsilon(\tau)) = \frac{d}{ds^n} \frac{1}{s} = \frac{(-1)^n (n+1)!}{s^{n+1}}$$

$$X_3(s) = \sum_{n=1}^{\infty} c_n \frac{(n+1)}{s^{n+1}} (-1)^n \quad = \sum \frac{c_n}{n!} \cdot \frac{(-1)^n (n+1)!}{s^{n+1}}$$

$$\underline{X(s)} = \mathcal{L}(x(\tau)) = 1 + 2 + 3 = \underline{\underline{\frac{c_0}{s} + \sum_{n=1}^{\infty} (-1)^n \left[c_n \frac{n+1}{s^{n+1}} + c_{-n} \frac{\mathcal{L}\left(\frac{\varepsilon(\tau)}{\tau^n}\right)}{n!}\right]}}$$

eg: sinusförmig eingeschwungener Zustand des Netzwerkes

$I_1 = I_2 = 10A$ bei $U = 100V$; $f = 50Hz$ $R_1 = 6\Omega$; $R_2 = 8\Omega$



Ges: a) L, C

b) I mit kompl. Rechnung; $\varphi(U,I)$

$Z_1 = R_1 + j\omega L$ $|Z_1| = \sqrt{R_1^2 + (\omega L)^2} = \frac{U}{I} = \frac{100V}{10A} = 10\Omega$

$Z_2 = R_2 + \frac{1}{j\omega C}$ $|Z_2| = \sqrt{R_2^2 + \left(\frac{1}{\omega C}\right)^2} = \frac{U}{I} = \frac{100V}{10A} = 10\Omega$

$$\sqrt{R_1^2 + (\omega L)^2} = 10\Omega \Rightarrow (\omega L)^2 = 100 - R_1^2 \Rightarrow \underline{\underline{L}} = \sqrt{100 - R_1^2} \cdot \frac{1}{\omega} = \underline{\underline{25{,}4\,mH}}$$

$$\sqrt{R_2^2 + \frac{1}{(\omega C)^2}} = 10\Omega \Rightarrow \left(\frac{1}{\omega C}\right)^2 = 100 - R_2^2 \Rightarrow \underline{\underline{C}} = \frac{1}{\omega\sqrt{100 - R_2^2}} = \underline{\underline{530{,}5\,\mu F}}$$

$$Z_g = Z_1 /\!/ Z_2 = \frac{(R_1 + j\omega L)(R_2 + \frac{1}{j\omega C})}{R_1 + R_2 + j(\omega L - \frac{1}{\omega C})} = \frac{R_1R_2 + L/C + j(\omega L R_2 - \frac{R_1}{\omega C})}{R_1 + R_2 + j(\omega L - \frac{1}{\omega C})}$$

$$= \frac{48 + 48 + j(64 - 36)}{6 + 8 + j(8-6)} = \frac{96 + j28}{14 + j2} = \frac{\sqrt{96^2 + 28^2} \cdot e^{j\varphi_1}}{\sqrt{14^2 + 2^2}\, e^{j\varphi_2}} = \frac{\sqrt{10000}}{\sqrt{200}}\, e^{j\varphi_1 - \varphi_2}$$

$\varphi_1 = \mathrm{atn}\,\frac{28}{96} = 16{,}26^\circ$ $\varphi_2 = \mathrm{atn}\,\frac{2}{14} = 8{,}13$

$$\Rightarrow \underline{Z_g} = \sqrt{50} \cdot e^{j16{,}26^\circ - 8{,}13^\circ} = \underline{7{,}07\, e^{j8{,}13}}$$

$$\Rightarrow \underline{\underline{\varphi(U,I)}} = \varphi_1 - \varphi_2 = \underline{\underline{8{,}13^\circ}}$$

Spannung voreilend, da Z_g induktiv ($\varphi_2 > \emptyset$)

FET

5.2) An einer Reihenschaltung von $R = 100\Omega$, $L = 40{,}6\,mH$ und $C = 10\mu F$ liegt die Spannung $u = 100V \cdot \cos(\omega t) + 40V \cdot \cos(3\omega t - \pi/6)$; $\omega = 2\pi 50Hz$

1) Berechnen Sie den Zeitverlauf des Stromes und seinen Effektivwert.

2) Geben Sie die aufgenommene Wirkleistung an.

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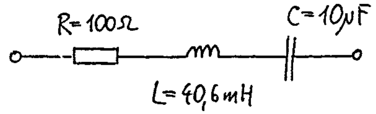
1) 

$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right) = 100 + j\left(\omega \cdot 0{,}0406 - \frac{1}{\omega \cdot 10 \cdot E\text{-}6}\right)\Omega$$

$$u = 100V \cdot \mathrm{Re}\{e^{j\omega t}\} + 40V \cdot \mathrm{Re}\{e^{j3\omega t} \cdot e^{-j\pi/6}\} = u_1 + u_2$$

$$i = i_1 + i_2$$

$$i_1 = \hat{i}_1 \cdot \mathrm{Re}\{e^{j\omega t} \cdot e^{j\varphi_1}\} = \frac{u_1}{Z(j\omega)} = \frac{100\ \mathrm{Re}\{e^{j\omega t}\}}{100 + j(2\pi 50 \cdot 0{,}0406 - \frac{1}{2\pi 50 \cdot 10 \cdot 10^{-6}})}A = \frac{\mathrm{Re}\{e^{j\omega t}\}100\,A}{100 + j(-305{,}5)}$$

$$= \mathrm{Re}\{e^{j\omega t}\} \cdot \frac{100}{\sqrt{100^2 + 305{,}5^2}} \cdot e^{-j\,\mathrm{atn}\frac{-305{,}5}{100}} = \frac{100}{322}\, e^{j1{,}25} = 0{,}311\, e^{j1{,}25}\, \mathrm{Re}\{e^{j\omega t}\}$$

$$= 0{,}311 \cos(\omega t + 1{,}25)A$$

$$i_2 = \hat{i}_2 \cdot \mathrm{Re}\{e^{j3\omega t} e^{j\varphi_2}\} = \frac{u_2}{Z(j3\omega)} = \frac{40\ \mathrm{Re}\{e^{j3\omega t}\} \cdot \mathrm{Re}\{e^{-j\pi/6}\}}{100 + j(6\pi 50 \cdot 0{,}0406 - \frac{1}{6\pi 50 \cdot 10 \cdot 10^{-6}})} = \frac{40 \cdot \mathrm{Re}\{e^{j3\omega t}\}}{100 - j67{,}84}A$$

$$= 40A \cdot \mathrm{Re}\left\{\frac{40\, e^{-j\pi/6}}{100 - j67{,}84} \cdot e^{j3\omega t}\right\} = 40A \cdot \mathrm{Re}\left\{\frac{40\, e^{-j\pi/6}}{120{,}8} \cdot e^{j0{,}6}\right\}$$

$$= 0{,}331 \cos(3\omega t + 0{,}6 - \pi/6)$$

$$\underline{\underline{I_{eff}}} = \sqrt{I_{1eff}^2 + I_{2eff}^2} = \frac{1}{\sqrt{2}}\sqrt{\hat{i}_1^2 + \hat{i}_2^2} = \frac{1}{\sqrt{2}}\sqrt{0{,}311^2 + 0{,}331^2} = \underline{\underline{0{,}321\,A}}$$

2) $\underline{\underline{P}} = I^2 \cdot R = 0{,}321^2 \cdot 100 = \underline{\underline{10{,}37\,W}}$

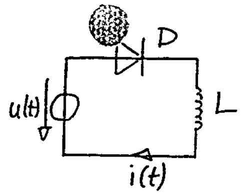
ad1) $\underline{i} = i_1 + i_2 = \underline{\underline{0{,}311\,A \cdot \cos(\omega t + 1{,}25) + 0{,}331\,A \cos(\ \omega t + 0{,}6 - \frac{\pi}{6})}}$

eg: • Schaltung mit idealer Diode

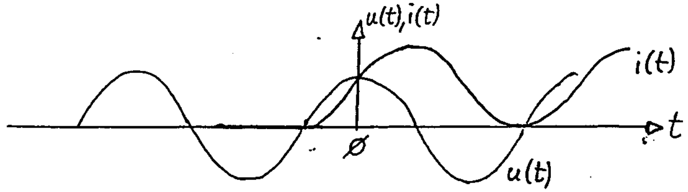
• $u(t) = U_e \cdot \sqrt{2} \cdot \cos(\omega t)$

es • $i(t)$ im eingeschwungenen Zustand

• P, S



, i(t)



$t < \frac{\pi}{2} \rightarrow u_{sin}(t) < Ø;\ i(t) = Ø$

$t \geq -\frac{\pi}{2} \rightarrow u_{sin}(t) \geq Ø \rightarrow i_{SIN}(t) = \frac{1}{L}\int_Ø^t u_{sin}(t)\,dt \quad > Ø$

$$i(t) = \frac{1}{L}\int U_e\sqrt{2}\cos(\omega t)\,dt = \frac{U_e\sqrt{2}}{\omega L}\sin(\omega t) + C \qquad i(t) \geq Ø$$

$$\text{RB: } i(t) = Ø \rightarrow C = -\frac{U_e\sqrt{2}}{\omega L}\sin(\omega t)\Big|_{\omega t = \frac{3\pi}{2}} = \frac{U_e\sqrt{2}}{\omega L}$$

$$\Rightarrow \underline{\underline{i(t) = \frac{U_e\sqrt{2}}{\omega L}\left(\sin(\omega t) + 1\right)}} \quad = i_1(t) + I_0$$

$$S = U \cdot I$$

$$U = \frac{\hat{U}}{\sqrt{2}} = \frac{U_e\sqrt{2}}{\sqrt{2}} = U_e$$

$$\underline{I} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(t)\,d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_Ø^{2\pi}\left(\frac{U_e\sqrt{2}}{\omega L}\right)^2 \cdot (1 + \sin(\omega t))^2\,d\omega t} = \sqrt{I_0^2 + I_1^2}$$

$$= \frac{U_e\sqrt{2}}{\omega L} \cdot \sqrt{\frac{1}{2\pi}\int_0^{2\pi} d(\omega t) + \frac{1}{2\pi}\int_0^{2\pi}\sin^2(\omega t)\,d\omega t} = \frac{U_e\sqrt{2}}{\omega L} \cdot \sqrt{\frac{3}{2}} = \underline{\frac{\sqrt{3}\,U_e}{\omega L}}$$

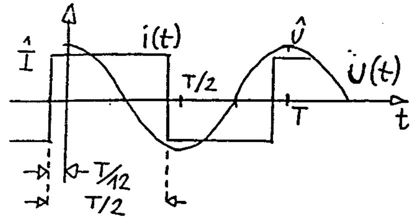
$$\underline{\underline{S = U \cdot I = \frac{\sqrt{3}\,U_e^2}{\omega L}}}$$

$$P = U \cdot I_1 \cdot \cos\varphi_1 + U \cdot I_0$$

φ_1 ... Winkel zwischen U und I_1 $\rightarrow \varphi_1 = 90° \rightarrow \cos\varphi_1 = Ø$

$$\underline{\underline{P}} = U \cdot I_0 = U_e \cdot \frac{U_e\sqrt{2}}{\omega L} = \underline{\underline{\frac{\sqrt{2}\,U_e^2}{\omega L}}}$$

5.6) Geg: Berechnen Sie die Grundschwingungsblindleistung zu der angegebenen Sinusspg und dem rechteckförmigen Wechselstrom.



$$u(t) = \hat{U}\cos(\omega t)$$

$$i(t) = \hat{I}\,\text{rect}\left(\frac{t - T/12}{T/2}\right) = \sum_{k=-\infty}^{\infty} c_{Ik}\,e^{jk\omega t} \qquad c_{Ik} \text{ ... } \mathcal{F}\text{-Reihe + Verschiebung}$$

$$i_1(t) = \text{Re}\left\{c_{1I}\,e^{j\omega t}\right\}$$

$$c'_{1I} = \frac{1}{T}\int i'(t)e^{-j\omega t}dt = \frac{1}{T}\left[\int_Ø^{T/2}\hat{I}e^{-j\omega t}dt + \int_{T/2}^{T}(-\hat{I})e^{-j\omega t}dt\right] = \frac{\hat{I}\cdot(e^{-j\omega\frac{T}{2}} - 1)}{-j\omega \cdot T} - \frac{\hat{I}(e^{-j\omega T} - e^{-j\omega\frac{T}{2}})}{-j\omega T}$$

$$= \frac{\hat{I}}{-j\omega T}\cdot\left(-e^{-j\omega T} + 2e^{-j\omega\frac{T}{2}} - 1\right) \overset{\omega = \frac{2\pi}{T}}{=} \frac{\hat{I}}{j2\pi}\cdot\left(\underbrace{e^{-j2\pi}}_{=1} - 2\underbrace{e^{-j\pi}}_{=-1} + 1\right) = \frac{2\hat{I}}{j\pi}$$

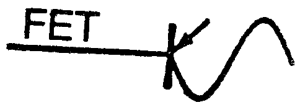
$$\underline{c_{1I} = e^{-j\pi/3}\cdot\frac{2\hat{I}}{j\pi}}$$

$$i_1(t) = \text{Re}\left\{\frac{2\hat{I}}{j\pi}\cdot e^{-j\pi/3}\cdot e^{j\omega t}\right\} = \frac{2\hat{I}}{\pi}\cdot\cos\left(\omega t - \pi/3\right) = \frac{2\hat{I}}{\pi}\sin\left(\omega t + \frac{\pi}{6}\right)$$

Scheinleistung der Grundschwingung: $S_1 = U \cdot |I_1| = \frac{\hat{U}}{\sqrt{2}} \cdot \frac{2\hat{I}}{\sqrt{2}\pi} = \frac{\hat{U}\hat{I}}{\pi}$

Grundschwingungsblindlstg: $Q_1 = S_1 \cdot \sin\varphi_1 = S_1\sin(\varphi_u - \varphi_i)$

$$\underline{\underline{Q_1}} = \frac{\hat{U}\cdot\hat{I}}{\pi}\cdot\sin\left(Ø - (-\pi/3)\right) = \underline{\underline{\frac{\hat{U}\hat{I}}{\pi}\sin(\pi/3)}}$$



eg: $X(s) = \frac{e^{-s}}{s^2} + \frac{e^{-2s}}{(s+1)^3}$

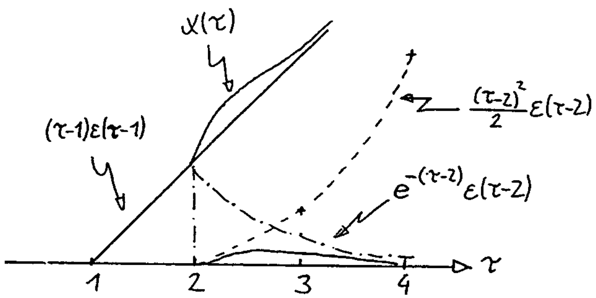
es: Graphische Darstellung von $x(\tau) \circ\!\!-\!\!\bullet X(s)$

geln: $X(s)e^{-s\tau_0} \bullet\!\!-\!\!\circ\ x(\tau-\tau_0)$

$\frac{1}{s^2} \bullet\!\!-\!\!\circ\ \tau\varepsilon(\tau)$

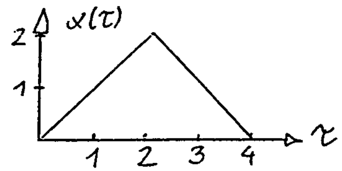
$\frac{1}{(s+1)^3} \bullet\!\!-\!\!\circ\ \frac{\tau^2}{2!}e^{-\tau}\varepsilon(\tau)$

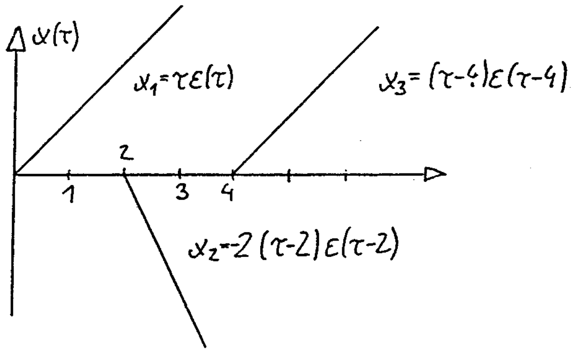
$$\underline{\underline{x(\tau) = \mathcal{L}(X(s)) = (\tau-1)\varepsilon(\tau-1) + \frac{(\tau-2)^2}{2}e^{-(\tau-2)}\varepsilon(\tau-2)}}$$



6.2) Geg: $x(\tau)$

Ges: $\mathcal{L}(x(\tau))$





$$x(\tau) = x_1 + x_2 + x_3 = \tau\varepsilon(\tau) - 2(\tau-2)\varepsilon(\tau-2) + (\tau-4)\varepsilon(\tau-4)$$

$$\underline{\underline{X(s)}} = \mathcal{L}(x(\tau)) = \frac{1}{s^2} - 2\frac{e^{-2s}}{s^2} + \frac{e^{-4s}}{s^2}$$

$$= \frac{1}{s^2}\cdot(1 - 2e^{-2s} + e^{-4s}) = \underline{\underline{\frac{1}{s^2}(1-e^{-2s})^2}}$$

FET

g: $x_1(\tau)=\cosh(\alpha\tau)\,\varepsilon(\tau)$

$x_2(\tau)=\sinh(\alpha\tau)\,\varepsilon(\tau)$

s: $\mathcal{L}(x_1(\tau))\,;\ \mathcal{L}(x_2(\tau))$

$$\cosh x=\frac{e^{x}+e^{-x}}{2}$$

$$x_1(\tau)=\frac{e^{\alpha\tau}+e^{-\alpha\tau}}{2}\cdot\varepsilon(\tau)$$

$$\underline{X_1(s)}=\mathcal{L}\{x_1(\tau)\}=\frac{1}{2}\left[\int_0^\infty e^{\alpha\tau}e^{-s\tau}d\tau+\int_0^\infty e^{-\alpha\tau}e^{-s\tau}d\tau\right]$$

$$=\frac{1}{2}\left[\frac{e^{(\alpha-s)\tau}\Big|_0^\infty}{\alpha-s}+\frac{e^{(-\alpha-s)\tau}\Big|_0^\infty}{-\alpha-s}\right]=\frac{1}{2}\left[\frac{-1}{\alpha-s}+\frac{-1}{-\alpha-s}\right]$$

$$=\frac{1}{2}\,\frac{\alpha+s-\alpha+s}{s^2-\alpha^2}=\underline{\underline{\frac{s}{s^2-\alpha^2}}}$$

$$\sinh x=\frac{e^{x}-e^{-x}}{2}$$

$$\underline{\underline{X_2(s)}}=\mathcal{L}\left(\frac{e^{\alpha\tau}-e^{-\alpha\tau}}{2}\,\varepsilon(\tau)\right)=\frac{1}{2}\left[\int_0^\infty e^{\alpha\tau}e^{-s\tau}d\tau-\int_0^\infty e^{-\alpha\tau}e^{-s\tau}d\tau\right]$$

$$=\frac{1}{2}\left[\frac{-1}{\alpha-s}-\frac{-1}{-\alpha-s}\right]=\frac{\alpha+s+\alpha-s}{2\cdot(s^2-\alpha^2)}$$

$$=\underline{\underline{\frac{\alpha}{s^2-\alpha^2}}}$$

FET

6.4) Geg: Eine Funktion $x(\tau)$ heißt antiperiodisch, wenn für ein festes $\tau_1>0$ und alle τ die Beziehung

$$x(\tau+\tau_1)=-x(\tau)$$

gilt. Zeigen Sie, daß ihre Laplace-Transformation durch

$$\mathcal{L}[x(\tau)]=(1+e^{-s\tau_1})^{-1}\int_0^{\tau_1}x(\tau)\,e^{-s\tau}d\tau$$

gegeben ist.

$$x(\tau+\tau_1)=-x(\tau)$$

$$\Rightarrow x(\tau)=\sum_{k=0}^{\infty}(-1)^k\,x_1(\tau-k\tau_1)\qquad \text{mit } x_1(\tau)=x(\tau)\cdot[\varepsilon(\tau)-\varepsilon(\tau-\tau_1)]$$

$$\underline{x(\tau)}=\sum_{k=0}^{\infty}\delta(\tau-k\tau_1)*(-1)^k x_1(\tau)=\underline{\sum_{k=0}^{\infty}(-1)^k\delta(\tau-k\tau_1)*x_1(\tau)}$$

$$\mathcal{L}[x(\tau)]=\underbrace{\mathcal{L}\left[\sum_{k=0}^{\infty}(-1)^k\delta(\tau-k\tau_1)\right]}_{1}\cdot\underbrace{\mathcal{L}[x_1(\tau)]}_{2}$$

$$1\ \circ\!\!-\!\!\bullet\ \sum_{k=0}^{\infty}(-1)^k\mathcal{L}[\delta(\tau-k\tau_1)]=\sum_{k=0}^{\infty}(-1)^k e^{-sk\tau_1}$$

$$\sum_{k=0}^{\infty}(-1)^k a^k=1-a+a^2-a^3\ldots=\sum_{k=0}^{\infty}(a^2)^k-a\sum_{k=0}^{\infty}(a^2)^k=\frac{1}{1-a^2}-\frac{a}{1-a^2}=\frac{1}{1+a}$$

$$\Rightarrow\ 1=\frac{1}{1+e^{-s\tau_1}}$$

$$2=\int_0^{\tau_1}x(\tau)e^{-s\tau}d\tau$$

$$\underline{\underline{\mathcal{L}[x(\tau)]=\frac{1}{1+e^{-s\tau_1}}\int_0^{\tau_1}x(\tau)e^{-s\tau}d\tau}}$$

Geg: In der Signalanalyse wird die Signalklasse

$$\varphi_n(\tau) = e^{-\tau/2} L_n(\tau)\varepsilon(\tau) \qquad n = 0, 1, 2, \ldots$$

mit den Laguerre-Polynomen

$$L_n(\tau) = \frac{e^{\tau}}{n!} \cdot \frac{d^n}{d\tau^n}\left(\tau^n e^{-\tau}\right)$$ verwendet.

Ges: Bestimmen Sie die zugehörige Klasse der Laplace Transformierten $\Phi_n(s)$.

$x(\tau)e^{s_0\tau} \circ\!\!-\!\!\bullet\ X(s-s_0)$

$$\varphi_n(\tau) = e^{\tau/2} \cdot \frac{1}{n!}\frac{d}{d\tau^n}\left(\tau^n e^{-\tau}\right) = e^{\tau/2} x_1(\tau)$$

$$\Phi_n(s) = X_1(s - \tfrac{1}{2})$$

$\frac{d}{d\tau^n}x(\tau) \circ\!\!-\!\!\bullet\ s^n X(s) - \sum s^{n-1-k} x^{(k)}$

$$x_1(\tau) = \frac{d}{d\tau^n}\left(\frac{\tau^n e^{-\tau}}{n!}\right) = \frac{d}{d\tau^n}\left(x_2(\tau)\varepsilon(\tau)\right)$$

$$X_1(s) = s^n X_2(s) - \sum_{k=0}^{n-1} s^k x_2^{(n-1-k)}(0^-)$$

$$x_2(\tau) = \frac{\tau^n e^{-\tau}}{n!}\varepsilon(\tau) \circ\!\!-\!\!\bullet\ X_2(s) = \frac{d^n}{ds^n}\frac{1}{s+1} \cdot \frac{1}{n!} = \frac{(n+1)!}{(s+1)^{n+1}}\frac{(-1)^n}{n!} = \frac{(n+1)(-1)^n}{(s+1)^{n+1}}$$

$$\Rightarrow \Phi_n(s) = (s-\tfrac{1}{2})^n \cdot (n+1)(-1)^n \cdot \frac{1}{(s+\frac{1}{2})^{n+1}} - \sum_{k=0}^{n-1}(s-\tfrac{1}{2})^k x_2^{(n-1-k)}(0-)$$

$$\underline{\underline{\Phi_n(s) = (n+1)(-1)^n \frac{(s-\frac{1}{2})^n}{(s+\frac{1}{2})^{n+1}} - \sum_{k=0}^{n-1}(s-\tfrac{1}{2})^k x_2^{(n-1-k)}(0-)}}$$

$$\tau^n \varepsilon(\tau) \circ\!\!-\!\!\bullet\ \frac{d}{ds^n}\frac{1}{s} = \frac{(-1)^n(n+1)!}{s^{n+1}}$$

$$\frac{1}{n!}\tau^n\varepsilon(\tau) \circ\!\!-\!\!\bullet\ \frac{d}{ds^n}\frac{1}{s}\frac{1}{n!} = \frac{(-1)^n(n+1)!}{s^{n+1}}\frac{1}{n!} = \frac{(-1)^n(n+1)}{s^{n+1}}$$

$$\frac{1}{n!}\tau^n e^{-a\tau}\varepsilon(\tau) \circ\!\!-\!\!\bullet\ \frac{d}{ds^n}\frac{1}{s+1}\frac{1}{n!} = \frac{(-1)^n(n+1)!}{(s+1)^{n+1}}\frac{1}{n!} = \frac{(-1)^n(n+1)}{(s+1)^{n+1}}$$

G

6.8) Geg: $G(s) = \frac{(s+2)s^3}{(s+3)(s+1)^2}e^{-5s}$

Ges: $g(\tau)$

$$G(s) = \frac{(s+2)s^3}{(s+3)(s^2+2s+1)}e^{-5s} = \frac{s^4+2s^3}{s^3+5s^2+7s+3}e^{-5s}$$

$$s^4+2s^3 : s^3+5s^2+7s+3 = s-3$$
$$\underline{-s^4-5s^3-7s^2-3s}$$
$$0 - 3s^3-7s^2-3s$$
$$\underline{+3s^3+15s^2+21s+9}$$
$$0 \quad 8s^2+18s+9$$

$$G(s) = \left(s-3+\frac{8s^2+18s+9}{s^3+5s^2+7s+3}\right)e^{-5s} = \left(s-3+\frac{8s^2+18s+9}{(s+3)(s+1)^2}\right)e^{-5s}$$

$$\frac{8s^2+18s+9}{(s+3)(s+1)^2} = \frac{A}{s+3} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$$A = \frac{8s^2+18s+9}{(s+1)^2}\Big|_{s=-3} = \frac{72-54+9}{4} = \frac{27}{4}$$

$$C = \frac{8s^2+18s+9}{s+3}\Big|_{s=-1} = \frac{+8-18+9}{2} = -\frac{1}{2}$$

$$B = \frac{d}{ds}\left(\frac{8s^2+18s+9}{s+3}\right)\Big|_{s=-1} = \frac{(16s+18)(s+3)-s\cdot(8s^2+18s+9)}{(s+3)^2} = \frac{2\cdot 2+1\cdot(8-18+9)}{4} = \frac{3}{4}$$

$$G(s) = \left(s-3+\frac{27/4}{s+3}+\frac{5/4}{s+1}-\frac{0{,}5}{(s+1)^2}\right)e^{-5s}$$

$$g(\tau) = \mathcal{L}^{-1}(G(s)) = \delta^{(1)}(\tau-5) - 3\delta(\tau-5) + \left[\frac{27}{4}e^{-3(\tau-5)} + \frac{5}{4}e^{-(\tau-5)} - \frac{1}{2}(\tau-5)e^{-(\tau-5)}\right] \cdot \varepsilon(\tau-5)$$

$$\underline{\underline{g(\tau) = \delta^{(1)}(\tau-5) - 3\delta(\tau-5) + \frac{1}{4}\left[27e^{-3(\tau-5)} + 5e^{-(\tau-5)} - 2(\tau-5)e^{-(\tau-5)}\right]\cdot\varepsilon(\tau-5)}}$$

FET

Geg: • DGL des Systems $y' + 0{,}5y = 2u - u(\tau-1)$

Ges: • Sprungantwort $h(\tau)$ + Skizze

$$y' + 0{,}5y = 2u - u(\tau-1) \quad / \mathcal{L}$$

$$Y(s) - y(0^-) + 0{,}5\,Y(s) = 2\,U(s) - U(s)e^{-s}$$

$$\Rightarrow Y(s) = \frac{2-e^{-s}}{s+0{,}5}\,U(s) + \frac{y(0^-)}{s+0{,}5}$$

Sprungantwort: • $u(\tau) = \varepsilon(\tau) \circ\!\!-\!\!\bullet\, U(s) = \frac{1}{s}$

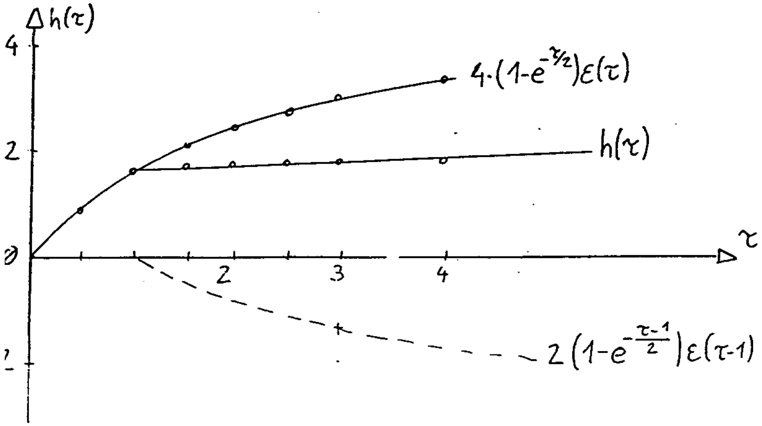
• $y(0^-) = 0$

$$H(s) = \frac{2-e^{-s}}{s+0{,}5}\cdot\frac{1}{s} = \left(\frac{A}{s} + \frac{B}{s+0{,}5}\right)\cdot(2-e^{-s})$$

$$A = \frac{1}{s+0{,}5}\Big|_{s=0} = 2 \qquad B = \frac{1}{s}\Big|_{s=-0{,}5} = -2$$

$$H(s) = (2-e^{-s})\left(\frac{2}{s} - \frac{2}{s+0{,}5}\right) = \frac{4}{s} - \frac{4}{s+0{,}5} - \frac{2e^{-s}}{s} + \frac{2e^{-s}}{s+0{,}5}$$

$$\underline{\underline{h(\tau) = \mathcal{L}(H(s)) = 4(1-e^{-\tau/2})\varepsilon(\tau) - 2(1-e^{-\frac{(\tau-1)}{2}})\varepsilon(\tau-1)}}$$



7.2) Die Beziehung zwischen der Eingangsgröße u und der Ausgangsgröße y eines Systems ist mit

$$y(\tau) = \sum_{n=-\infty}^{\infty} \delta\left(\tau - n\frac{T}{T_B}\right) u(\tau) \qquad \text{gegeben.}$$

Ist diese Beziehung und damit das System

1) Linear

2) zeitinvariant ? Begründen Sie jeweils vollständig.

1) $y(\tau) = y_{0E}(\tau) + y_{0Z}(\tau)$

$y_{0E}(\tau) = 0 \quad \Rightarrow y_{0Z}(\tau) = y(\tau) = \sum \delta(\tau - n\frac{T}{T_B})u(\tau)$

a) Linearität von y_{0E} bezüglich Anfangszustandes → $y_{0E} = 0$ → O.K.

b) Linearität von y_{0Z} bezüglich Eingangsfunktion $u(\tau) = \alpha_1 U_1(\tau) + \alpha_2 U_2(\tau)$

$$y_{0Z}(\tau) = \sum \delta(\tau - n\tfrac{T}{T_B})\,u(\tau) = \sum \delta(\tau - n\tfrac{T}{T_B})(\alpha_1 U_1(\tau) + \alpha_2 U_2(\tau)) \quad \Rightarrow \text{O.K.}$$
$$= \sum \delta(\tau - n\tfrac{T}{T_B})\,\alpha_1 U_1(\tau) + \alpha_2 \sum \delta(\tau - n\tfrac{T}{T_B})\,U_2(\tau) = \alpha_1 y_{0Z1}(\tau) + \alpha_2 y_{0Z2}(\tau)$$

a) und b) sind erfüllt ⇒ <u>System ist Linear</u>

2) Zeitvarianz:

Verschiebung von $u(\tau)$ um $\tau_0 \to u(\tau-\tau_0)$ ⇒ $y(\tau)$ muß $y(\tau-\tau_0)$ folgen

$$y(\tau-\tau_0) = \sum_{n=-\infty}^{\infty} \delta(\tau - n\tfrac{T}{T_B})\,u(\tau-\tau_0) \qquad \text{erfüllt?} \qquad \tau' = (\tau-\tau_0)$$

$$y(\tau') = \sum_{n=-\infty}^{\infty} \delta(\tau' + \tau_0 - n\tfrac{T}{T_B})\,u(\tau')$$

⇒ $\tau_0 = k\frac{T}{T_B}$ k ganze Zahl, auch neg. ⇒ System zeitinvariant

τ_0 = sonst System zeitvariant

weil τ_0 nicht beliebig sein kann ⇒ <u>System zeitvariant !</u>

g: •) DGL des Systems: $y''+4y'+3y=4u$

•) Eingangsfunktion: $u(\tau)=\delta(\tau)-2\varepsilon(\tau)$

•) $y'(0^-)=0$, $y(0^-)=0$, $u(0^-)=0$

es: $y_{0z}(\tau)$ im Originalbereich und im Bildbereich

$$+4y'+3y=4u \qquad /\mathcal{L}$$

$$Y(s)-sy(0^-)-y'(0^-)+4sY(s)-4y(0^-)+3Y(s)=4U(s)$$

$$Y(s)=\underbrace{\frac{4}{s^2+4s+3}U(s)}_{Y_{0z}(s)}+\underbrace{\frac{sy(0^-)+y'(0^-)+4y(0^-)}{s^2+4s+3}}_{Y_{0\varepsilon}(s)}$$

$$s)=\mathcal{L}(u(\tau))=\mathcal{L}(\delta(\tau)-2\varepsilon(\tau))=1-\frac{2}{s}=\frac{s-2}{s}$$

$$_z(s)=\frac{4}{s^2+4s+3}\cdot\frac{s-2}{s}=\frac{A}{(s+3)}+\frac{B}{(s+1)}+\frac{C}{s}$$

$$A=\frac{4(s-2)}{(s+1)\cdot s}\bigg|_{s=-3}=\frac{4\cdot(-5)}{(-2)\cdot(-3)}=\frac{-20}{6}=-\frac{10}{3}$$

$$B=\frac{4(s-2)}{(s+3)\cdot s}\bigg|_{s=-1}=\frac{4\cdot(-3)}{2\cdot(-1)}=\frac{-12}{-2}=6$$

$$C=\frac{4(s-2)}{s^2+4s+3}\bigg|_{s=0}=\frac{-8}{3}=-\frac{8}{3}$$

$$(s)=-\frac{10}{3}\frac{1}{(s+3)}+\frac{6}{s+1}-\frac{8}{3s}$$

$$(\tau)=\mathcal{L}^{-1}(Y_{0z}(s))=-\frac{10}{3}e^{-3\tau}\varepsilon(\tau)+6e^{-\tau}\varepsilon(\tau)-\frac{8}{3}\varepsilon(\tau)$$

$$=\left(-\frac{8}{3}-\frac{10}{3}e^{-3\tau}+6e^{-\tau}\right)\varepsilon(\tau)$$

FET

7.4) Geg: •) System $y'+5y=2u'+3u$

•) Eingang $u(\tau)=e^{-\tau}\varepsilon(\tau)$

•) Anfangswerte $y(0^-)=-5$; $u(0^-)=0$

Ges: •) $Y(s)$

•) $y_{0\varepsilon}(\tau)$, $y_{0z}(\tau)$, $y(\tau)$

$$y'+5y=2u'+3u \qquad |\mathcal{L}$$

FET

$$sY(s)-y(0^-)+5Y(s)=2sU(s)+3U(s)-2u(0^-)$$

$$\Rightarrow Y(s)=\frac{(2s+3)}{s+5}U(s)+\frac{y(0^-)-2u(0^-)}{s+5}=\underbrace{\frac{2s+3}{s+5}U(s)}_{Y_{0z}(s)}-\underbrace{\frac{5}{s+5}}_{Y_{0\varepsilon}(s)}$$

$$Y_{0\varepsilon}(s)=\frac{-5}{s+5} \quad \bullet\!\!-\!\!\circ \quad y_{0\varepsilon}(\tau)=-5e^{-5\tau}\varepsilon(\tau)$$

$$U(s)=\mathcal{L}(u(\tau))=\mathcal{L}(e^{-\tau}\varepsilon(\tau))=\frac{1}{s+1}$$

$$Y_{0z}(s)=\frac{2s+3}{(s+5)(s+1)}=\frac{A}{s+5}+\frac{B}{s+1}$$

$$A=\frac{2s+3}{s+1}\bigg|_{s=-5}=\frac{-7}{-4}=\frac{7}{4} \qquad B=\frac{2s+3}{s+5}\bigg|_{s=-1}=\frac{1}{4}$$

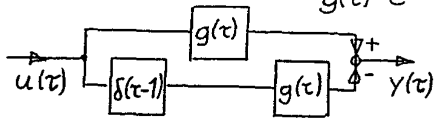
$$Y_{0z}(s)=\frac{1}{4}\left[\frac{7}{s+5}+\frac{1}{s+1}\right]$$

$$y_{0z}(\tau)=\mathcal{L}^{-1}(Y_{0z}(s))=\frac{1}{4}\left(7e^{-5\tau}+e^{-\tau}\right)\varepsilon(\tau)$$

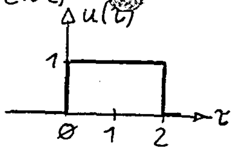
$$y(\tau)=y_{0\varepsilon}(\tau)+y_{0z}(\tau)=\left(\frac{7}{4}e^{-5\tau}+\frac{e^{-\tau}}{4}-5e^{-5\tau}\right)\varepsilon(\tau)$$

$$y(\tau)=\frac{1}{4}\left(e^{-\tau}-13e^{-5\tau}\right)\varepsilon(\tau)$$

Geg:



$g(\tau)=e^{-(\tau-2)}\varepsilon(\tau-2)$



Ges: Berechnen und skizzieren Sie für die angegebene Kombination den Ausgang y zum vorliegenden Eingang.

$$G(s)=\mathcal{L}(g(\tau))=\frac{e^{-2s}}{s+1}$$

$$G_1(s)=G(s)\cdot\mathcal{L}(\delta(\tau-1))=G(s)\cdot e^{-s}=\frac{e^{-3s}}{s+1}$$

$$G_{ges}(s)=G(s)-G_1(s)=\frac{1}{s+1}\left(e^{-2s}-e^{-3s}\right)$$

$$u(\tau)=\varepsilon(\tau)-\varepsilon(\tau-2)$$

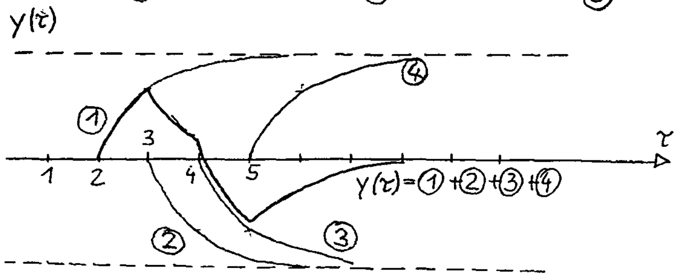
$$U(s)=\mathcal{L}(u(\tau))=\frac{1}{s}-\frac{e^{-2s}}{s}=\frac{1}{s}\left(1-e^{-2s}\right)$$

$$Y(s)=G_{ges}(s)\cdot U(s)=\frac{1}{s+1}\cdot\frac{1}{s}\cdot\left(e^{-2s}-e^{-3s}-e^{-4s}+e^{-5s}\right)$$

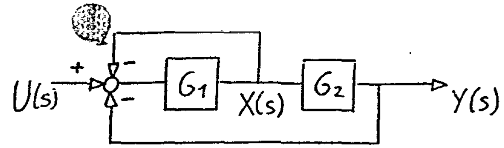
$$\left.\begin{array}{l}\frac{1}{s+1}\cdot\frac{1}{s}=\frac{A}{s+1}+\frac{B}{s}\\[1ex] A=\left.\frac{1}{s}\right|_{s=-1}=-1 \qquad B=\left.\frac{1}{s+1}\right|_{s=\varnothing}=1\end{array}\right\}\sim=\frac{1}{s}-\frac{1}{s+1}$$

$$Y(s)=\left(\frac{1}{s}-\frac{1}{s+1}\right)\left(e^{-2s}-e^{-3s}-e^{-4s}+e^{-5s}\right)=\left(\frac{1}{s}-\frac{1}{s+1}\right)e^{-2s}-\left(\frac{1}{s}-\frac{1}{s+1}\right)e^{-3s}-\ldots\ldots$$

$$y(\tau)=\underbrace{\left(1-e^{-(\tau-2)}\right)\varepsilon(\tau-2)}_{1}-\underbrace{\left(1-e^{-(\tau-3)}\right)\varepsilon(\tau-3)}_{2}-\underbrace{\left(1-e^{-(\tau-4)}\right)\varepsilon(\tau-4)}_{3}+\underbrace{\left(1-e^{-(\tau-5)}\right)\varepsilon(\tau-5)}_{4}$$



7.6) Geg:



Ges: $G(s)$

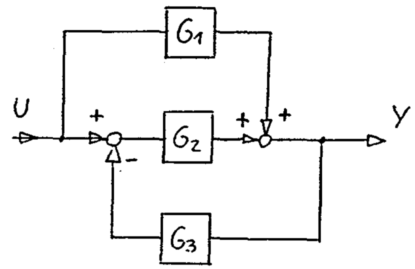
$$X(s)=G_1(s)\cdot\left(U(s)-X(s)-Y(s)\right)$$

$$\Rightarrow X(s)=\frac{G_1(s)}{1+G_1(s)}\cdot\left(U(s)-Y(s)\right)$$

$$Y(s)=G_2(s)X(s)=\frac{G_1(s)G_2(s)}{1+G_1(s)}\cdot\left(U(s)-Y(s)\right)$$

$$\Rightarrow Y(s)=\frac{\frac{G_1(s)\cdot G_2(s)}{1+G_1(s)}U(s)}{1+\frac{G_1(s)\cdot G_2(s)}{1+G_1(s)}}=\frac{G_1(s)\cdot G_2(s)}{1+G_1(s)+G_1(s)G_2(s)}U(s)$$

7.7) Geg:



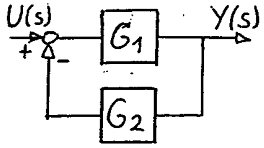
Ges: $G(s)$

$$Y(s)=G_1(s)U(s)+G_2(s)\cdot\left(U(s)-G_3(s)Y(s)\right)$$

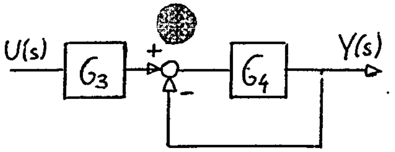
$$=\left[G_1(s)+G_2(s)\right]U(s)-G_2(s)\cdot G_3(s)Y(s)$$

$$\Rightarrow Y(s)=\frac{G_1(s)+G_2(s)}{1+G_2(s)G_3(s)}\cdot U(s)$$

Geg: a)



b)



Ges: $G_3 = f(G_1, G_2)$, $G_4 = f(G_1, G_2)$ für gleiche Übertragungsfunktionen G_{ges}

a) $Y(s) = G_{a,ges}(s) \cdot U(s) = G_1(s) \cdot (U(s) - G_2(s) \cdot Y(s))$

$$\Rightarrow G_{a,ges} = \frac{G_1(s)}{1 + G_1(s) G_2(s)}$$

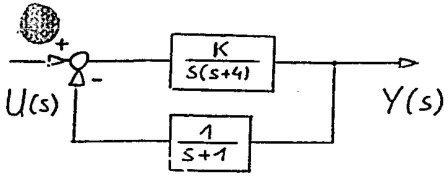
b) $Y(s) = G_{b,ges}(s) \cdot U(s) = G_3(s) \cdot G_4(s) \cdot U(s) - G_4(s) Y(s)$

$$\Rightarrow G_{b,ges} = \frac{G_3(s) G_4(s)}{1 + G_4(s)}$$

$G_{a,ges} = G_{b,ges} \Rightarrow \underline{G_4(s) = G_1(s) \cdot G_2(s)}$

$G_3(s) \cdot G_4(s) = G_1(s) \Rightarrow \underline{\underline{G_3(s)}} = \frac{G_1(s)}{G_1(s) \cdot G_2(s)} = \underline{\underline{\frac{1}{G_2(s)}}}$

7.9) Geg:



$K > 0$

Ges: •) K für Grenzstabilität

•) ϑ bei ——"——

•) $Y(s) = \frac{K}{s(s+4)} \left(U(s) - \frac{1}{s+1} \right) Y(s)$

$$\Rightarrow \underline{Y(s)} = \frac{\frac{K}{s(s+4)}}{1 + \frac{K}{s(s+4)(s+1)}} U(s) = \frac{K(s+1)\; U(s)}{K + s(s+4)(s+1)} = \underline{\frac{K(s+1)}{s^3 + 5s^2 + 4s + K} \cdot U(s)}$$

Grenzstabilität: einfacher Pol auf imaginären Achse:

Pole: $s^3 + 5s^2 + 4s + K = (s + p_1)(s + p_2)(s + p_3) = N(s)$

$N(0) = K \neq 0 \Rightarrow p_1 = j\omega_1 \; ; \; p_2 = -j\omega_1$

$(s + p_1)(s + p_2) = (s + j\omega_1)(s - j\omega_1) = (s^2 + \omega_1^2)$

$(s^3 + 5s^2 + 4s + K) : (s^2 + \omega_1^2) = s + 5$

$-s^3 \quad -\omega_1^2 s$

$0 \quad 5s^2 + s(4 - \omega_1^2) + K$

$-5s^2 \quad -5\omega_1^2$

$s(4 - \omega_1^2) + K - 5\omega_1^2 \underline{\underline{= 0}}$ für Grenzstab.

$\Rightarrow \omega_1^2 = 4 \Rightarrow \omega_1 = \pm 2$

$K - 5 \cdot 4 = 0 \Rightarrow \underline{\underline{K = 20}}$

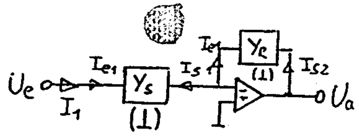
$$G(s) = \frac{20(s+1)}{(s^2 + 4)(s + 5)}$$

•) $\omega_{1,2} = \pm 2 \Rightarrow (s + j2)(s - j2) = (s^2 + 0 \cdot s + 4) = s^2 + 2\vartheta \cdot \omega_1 \cdot s + \omega_1^2$

$\Rightarrow \vartheta = 0$

d.h., keine Dämpfung für Resonanzfrequenz $\omega_{1,2}$

Geg: OPV-Verstärker

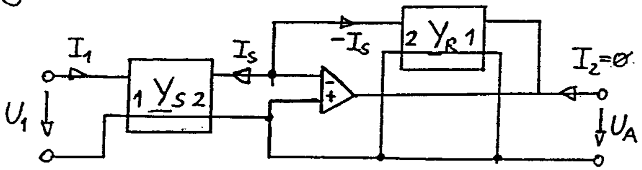


Ges: Übertragungsfunktion $G(j\omega)$ für

a) geerdete Y; d.h., $I_e \neq I_s$

b) nicht geerdete Y; d.h., $I_e = I_s$

a) geerdete Admitanzen $\Rightarrow$ Y sind Zweitore (Vierpole)



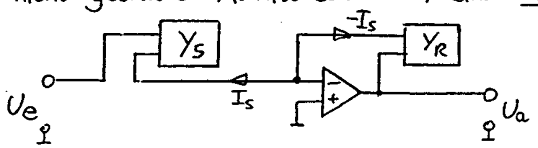
allgemein: $\underline{I} = \underline{Y} \cdot \underline{U}$ $\Rightarrow$ $I_2 = Y_{21} \cdot U_1 + Y_{22} \cdot U_2$ (Vierpol)

Y_S: $\quad I_s = Y_{21S} \cdot U_e + Y_{22S} \cdot \emptyset \quad (U_{ed} = \emptyset)$

Y_R: $\quad -I_s = Y_{21R} \cdot U_a + Y_{22R} \cdot \emptyset$

$$\Rightarrow I_s = Y_{21S} U_e = -Y_{21R} U_a \Rightarrow \underline{\underline{G(j\omega) = \frac{U_a(j\omega)}{U_e(j\omega)} = -\frac{Y_{21S}}{Y_{21R}}}}$$

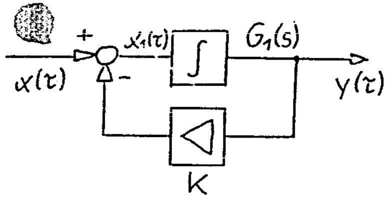
b) nicht geerdete Admitanzen $\Rightarrow$ Y sind Zweipole



$I_s = -U_e \cdot Y_S \qquad -I_s = -U_a Y_R$

$$\Rightarrow I_s = -U_e Y_S = U_a Y_R \Rightarrow \underline{\underline{G(j\omega) = \frac{U_a(j\omega)}{U_e(j\omega)} = -\frac{Y_S}{Y_R}}}$$

7.11) Geg:



Ges: • Verstärkung K für stabilen Bereich

• Sprungantwort $h(\tau)$

• $G_1(s)$: $\quad y(\tau) = g_1(\tau) * x_1(\tau) = \int_\emptyset^\tau x_1(\tau')d\tau' = \int_\emptyset^\infty x_1(\tau') \cdot \varepsilon(\tau-\tau')d\tau'$

$$= \int_\emptyset^\infty x_1(\tau') g_1(\tau-\tau')d\tau'$$

$$\Rightarrow \varepsilon(\tau-\tau') = g_1(\tau-\tau') \Rightarrow g_1(\tau) = \varepsilon(\tau) \circ\!\!-\!\!\bullet\ G_1(s) = \frac{1}{s}$$

System: $\quad Y(s) = G_1(s) \cdot (X(s) - K\,Y(s)) = G_1(s)X(s) - G_1(s)K\,Y(s)$

$$\Rightarrow \underline{Y(s)} = \frac{G_1(s)}{1+K\,G_1(s)} X(s) = \frac{1/s}{1+K/s} X(s) = \underline{\frac{X(s)}{s+K}}$$

Stabilität: Pole auf linker Halbebene $\Rightarrow$ $\underline{\underline{\mathrm{Re}(K) > \emptyset}}$

• $\quad x(\tau) = \varepsilon(\tau) \circ\!\!-\!\!\bullet\ X(s) = \frac{1}{s}$

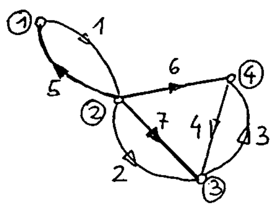
$$H(s) = \frac{1}{s+K} \cdot \frac{1}{s} = \frac{A}{s} + \frac{B}{s+K}$$

$$A = \left.\frac{1}{s+K}\right|_{s=\emptyset} = \frac{1}{K} \qquad B = \left.\frac{1}{s}\right|_{s=-K} = -\frac{1}{K}$$

$$H(s) = \frac{1}{K} \cdot \left(\frac{1}{s} - \frac{1}{s+K}\right) \bullet\!\!-\!\!\circ\ \underline{\underline{h(\tau) = \frac{1}{K} \cdot (1-e^{-K\tau})\,\varepsilon(\tau)}}$$

FET

eg:



es: M, S, A_v, A

Maschenmatrix M

Maschen mit 1em Verbindungszweig mit dessen Orientierung:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix}$$

Dim: Maschen x Zweige

Schnittmatrix S

Schnitte so, daß immer nur 1 Baumzweig (legt Orientierung fest) betroffen ist.

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Dim: Baumzweige x Zweige

Inzidenzmatrix A_v

hin- & wegführende Zweige zu Knoten → Dim: Knoten x Zweige

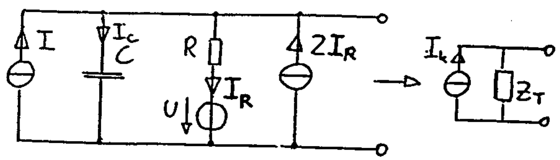
$$A_v = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 & 0 \end{bmatrix}$$

eduzierte Inzidenzmatrix A:

Zeile des Bezugpotentials streichen:

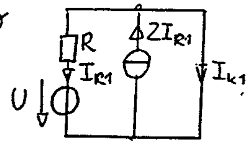
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 & 0 \end{bmatrix}$$

8.2) Geg: Schaltung.
Ges: I_k, Z_T



I_k: Superposition: nur unabhängige Quellen entfernen!

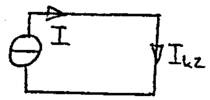
a) $I = \emptyset$



$2I_{R1} = I_{R1} + I_{k1}$

$\underline{I_{k1}} = I_{R1} = \underline{-\frac{U}{R}}$

b) $U = \emptyset$



$\underline{I_{k2} = I}$

$$\underline{\underline{I_k}} = I_{k1} + I_{k2} = \underline{\underline{I - \frac{U}{R}}}$$

U_L: Knotenregel: $I - I_C - I_R + 2I_R = \emptyset \Rightarrow I + I_R = I_C$ (1)

$U_L = I_C \cdot \frac{1}{sC}$ (2)

$U_L = U + I_R \cdot R$ (3)

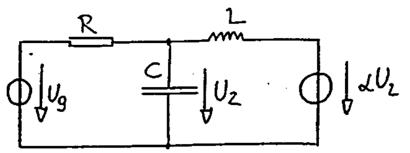
$\Rightarrow I_C$ & I_R eliminieren

(2),(3) in (1): $I + \frac{U_L}{R} - \frac{U}{R} = sCU_L \Rightarrow \underline{U_L = \frac{I - U/R}{sC - 1/R}}$

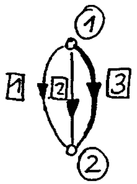
$$\underline{\underline{Z_T}} = \frac{U_L}{I_K} = \frac{I - U/R}{sC - 1/R} \cdot \frac{1}{I - U/R} = \underline{\underline{\frac{1}{sC - 1/R}}}$$

FET

Geg:



Ges: Netzwerk, $\underset{\sim}{Z}$, $\underset{\sim}{I_q}$, $\underset{\sim}{U_q}$



$$\rightarrow \quad I_q = \begin{pmatrix} \varnothing \\ \varnothing \\ \frac{i_3(\varnothing)}{s} \end{pmatrix} \quad U_q = \begin{pmatrix} U_g \\ \frac{u_c(\varnothing^-)}{s} \\ \varnothing \end{pmatrix}$$

$$U_g = \varnothing \rightarrow \quad U_1' = R \cdot I_1$$

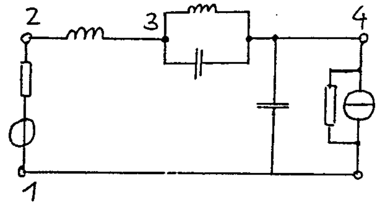
$$U_2' = \frac{1}{j\omega C} \cdot I_2$$

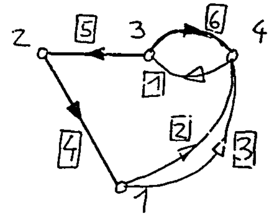
$$U_3' = j\omega L \cdot I_3 + \alpha U_2 = j\omega L\, I_3 + \frac{\alpha}{j\omega C} I_2$$

$$\underline{\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} R & \varnothing & \varnothing \\ \varnothing & \frac{1}{j\omega C} & \varnothing \\ \varnothing & \frac{\alpha}{j\omega C} & j\omega L \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}} + \begin{pmatrix} U_g \\ \frac{u_c(\varnothing-)}{s} \\ \varnothing \end{pmatrix} - \underset{\sim}{Z} \cdot \begin{pmatrix} \varnothing \\ \varnothing \\ \frac{i_L(\varnothing_-)}{s} \end{pmatrix}$$

$$\underline{\underline{\underset{\sim}{Z} = \begin{pmatrix} R & \varnothing & \varnothing \\ \varnothing & 1/j\omega C & \varnothing \\ \varnothing & \alpha/j\omega C & j\omega L \end{pmatrix}}}$$

8.4) Geg:





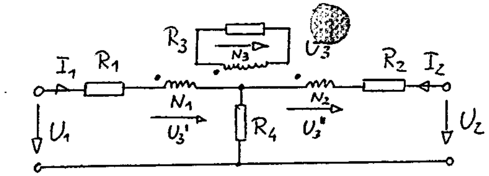
Bestimmen Sie für das Netzwerk mit Hilfe des angegebenen Graphen und des darin festgelegten Baumes die Schnittmatrix S und die Maschenmatrix M.

$$S = \begin{bmatrix} 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \qquad M = \begin{bmatrix} 1 & \varnothing & \varnothing & \varnothing & \varnothing & 1 \\ \varnothing & 1 & \varnothing & 1 & 1 & -1 \\ \varnothing & \varnothing & 1 & 1 & 1 & -1 \end{bmatrix}$$

$$u = \begin{pmatrix} U_v \\ U_a \end{pmatrix} \qquad S = (S_v \;\; E) \qquad M = (E \;\; M_B) \qquad S_v = -M_B^T$$

Geg: Schaltung

Ges: $\underset{\sim}{Z}$-Matrix



$$U_1 = I_1 \cdot R_1 + U_3' + R_4(I_1+I_2)$$

$$U_2 = I_2 \cdot R_2 - U_3'' + R_4(I_1+I_2)$$

$$U_3 = R_3 \cdot \left(I_2 \cdot \frac{N_2}{N_3} - I_1 \frac{N_1}{N_3}\right) \Rightarrow \begin{cases} U_3' = \frac{N_1}{N_3} U_3 \\ U_3'' = \frac{N_2}{N_3} U_3 \end{cases}$$

$$\Rightarrow U_1 = R_1 I_1 + \frac{N_1}{N_3} \cdot I_2 \cdot \frac{N_2}{N_3} \cdot R_3 - \frac{N_1}{N_3} \cdot I_1 \cdot \frac{N_1}{N_3} \cdot R_3 + R_4 I_1 + R_4 I_2$$

$$= I_1 \cdot \left(R_1 + R_4 - \left(\frac{N_1}{N_3}\right)^2 \cdot R_3\right) + I_2 \left(R_4 + \frac{N_1 N_2}{N_3^2} R_3\right)$$

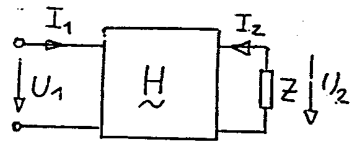
$$U_2 = I_2 R_2 - \frac{N_2}{N_3} \cdot R_3 \cdot I_2 \frac{N_2}{N_3} + \frac{N_2}{N_3} \cdot R_3 I_1 \frac{N_1}{N_3} + R_4 I_1 + R_4 I_2$$

$$= I_1 \cdot \left(R_4 + R_3 \frac{N_1 N_2}{N_3^2}\right) + I_2 \cdot \left(R_2 + R_4 - \frac{N_1 N_2}{N_3^2} R_3\right)$$

$$\Rightarrow \underline{\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} R_1 + R_4 - R_3\left(\frac{N_1}{N_3}\right)^2 & R_4 + R_3 \frac{N_1 N_2}{N_3^2} \\ R_4 + R_3 \frac{N_1 N_2}{N_3^2} & R_2 + R_4 - \frac{N_1 N_2}{N_3^2} R_3 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}}$$

8.6) Geg: Zweitor, beschrieben mit der Hybridmatrix $\underset{\sim}{H}$, mit Abschluß Z

Ges: $U_2 = f(U_1)$



$$\begin{pmatrix} U_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ U_2 \end{pmatrix} \rightarrow \begin{aligned} U_1 &= H_{11} I_1 + H_{12} U_2 && (1) \\ I_2 &= H_{21} I_1 + H_{22} U_2 && (2) \\ U_2 &= -I_2 \cdot Z && (3) \end{aligned}$$

aus (1) $I_1 = \frac{1}{H_{11}} (U_1 - H_{12} U_2)$

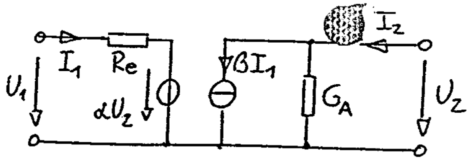
in (2) $I_2 = \frac{H_{21}}{H_{11}} (U_1 - H_{12} U_2) + H_{22} U_2$

in (3) $U_2 = Z\left[\frac{H_{21}}{H_{11}} \cdot H_{12} U_2 - \frac{H_{21}}{H_{11}} U_1 - H_{22} U_2\right]$

$$U_2 \cdot \left[\frac{1}{Z} + H_{22} - \frac{H_{21}}{H_{11}} \cdot H_{12}\right] = -\frac{H_{21}}{H_{11}} \cdot U_1$$

$$\underline{\underline{U_2}} = \frac{-H_{21} \cdot U_1}{\frac{H_{11}}{Z} + H_{11} \cdot H_{22} - H_{12} \cdot H_{21}} = \underline{\underline{\frac{-H_{21} \cdot U_1}{\frac{H_{11}}{Z} + \det(\underset{\sim}{H})}}}$$

Geg: Schaltung



Ges: •) Hybridmatrix

•) Betriebsimpetanzen $Z_{B1}(R_2)$, $Z_{B2}(R_1)$

Schaltung: ① $U_1 = I_1 \cdot R_e + \alpha U_2$

② $I_2 = \beta I_1 + I_{GA} = \beta I_1 + G_A U_2$

Hybridform: $\begin{pmatrix} U_1 \\ I_2 \end{pmatrix} = \underset{\sim}{H} \begin{pmatrix} I_1 \\ U_2 \end{pmatrix} \Rightarrow \begin{array}{l} U_1 = H_{11} I_1 + H_{12} \cdot U_2 \\ I_2 = H_{21} I_1 + H_{22} \cdot U_2 \end{array}$

$\Rightarrow$ Vgl. $\Rightarrow$ $\underset{\sim}{H} = \begin{pmatrix} R_e & \alpha \\ \beta & G_A \end{pmatrix}$

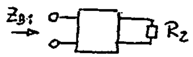
Betriebsimpetanzen:

a) $Z_{B1}(R_2) \Rightarrow$ Abschluß des Tores 2 mit $R_2 \Rightarrow U_2 = -I_2 \cdot R_2$

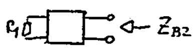
② $\Rightarrow I_2 = \beta I_1 + G_A U_2 = -\frac{U_2}{R_2} \Rightarrow U_2 = \frac{\beta I_1}{-G_A - 1/R_2}$

in ① $\Rightarrow U_1 = R_e I_1 + \frac{\alpha \cdot \beta \cdot I_1}{-G_A - 1/R_2} = I_1 \cdot \left(R_e - \frac{\alpha \cdot \beta}{G_A + 1/R_2}\right)$

$\underline{\underline{Z_{B1}(R_2) = \frac{U_1}{I_1} = R_e - \frac{\alpha \cdot \beta}{G_A + 1/R_2}}}$



b) $Z_{B2}(R_1) \Rightarrow$ Tor 1 mit R_1 abschließen

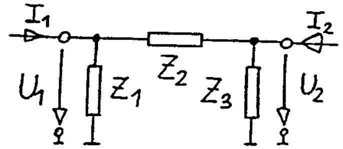


$U_1 = -I_1 \cdot R_1 = I_1 \cdot R_e + \alpha U_2 \Rightarrow I_1 = \frac{\alpha U_2}{-R_1 - R_e}$ mit ①

in ② $I_2 = \beta \frac{\alpha}{-R_1 - R_e} U_2 + G_A U_2 = U_2 \left(G_A - \frac{\alpha \cdot \beta}{R_1 + R_e}\right)$

$\underline{\underline{Z_{B2}(R_1) = \frac{U_2}{I_2} = \frac{1}{G_A - \frac{\alpha \cdot \beta}{R_1 + R_e}}}}$

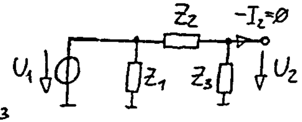
8.8) Geg: Vierpol



Ges: Kettenmatrix $\underset{\sim}{A}$

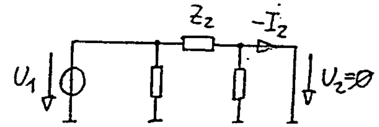
$\begin{pmatrix} U_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} U_2 \\ -I_2 \end{pmatrix} \Rightarrow \begin{array}{l} U_1 = A_{11} U_2 - A_{12} I_2 \\ I_1 = A_{21} U_2 - A_{22} I_2 \end{array}$

$A_{11} = \frac{1}{U_2}(U_1 + A_{12} I_2) = \frac{U_1}{U_2}\Big|_{I_2 = 0}$



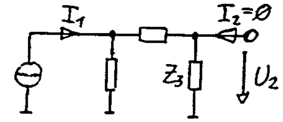
$U_2 = \frac{Z_3}{Z_2 + Z_3} U_1 \Rightarrow \underline{A_{11} = \frac{Z_2 + Z_3}{Z_3}}$

$A_{12} = \frac{U_1}{-I_2}\Big|_{U_2 = 0}$



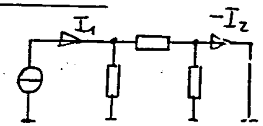
$-I_2 = \frac{U_1}{Z_2} \Rightarrow \underline{A_{12} = Z_2}$

$A_{21} = \frac{I_1}{U_2}\Big|_{I_2 = 0}$



$U_2 = I_{23} \cdot Z_3 = \frac{Z_1}{Z_1 + Z_2 + Z_3} \cdot I_1 \cdot Z_3 \Rightarrow \underline{A_{21} = \frac{Z_1 + Z_2 + Z_3}{Z_1 \cdot Z_3}}$

$A_{22} = \frac{I_1}{-I_2}\Big|_{U_2 = 0}$



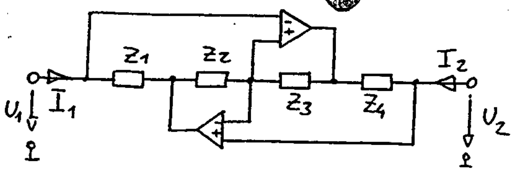
$-I_2 = I_1 \cdot \frac{Z_1}{Z_1 + Z_2} \Rightarrow \underline{A_{22} = \frac{Z_1 + Z_2}{Z_1}}$

$\underline{\underline{\underset{\sim}{A} = \begin{pmatrix} \frac{Z_2 + Z_3}{Z_3} & Z_2 \\ \frac{Z_1 + Z_2 + Z_3}{Z_1 \cdot Z_3} & \frac{Z_1 + Z_2}{Z_1} \end{pmatrix}}}$

Probe: passiver VP $\rightarrow$ reziprok $\rightarrow$ $\underline{\det A = 1} = \frac{Z_2 + Z_3}{Z_3} \cdot \frac{Z_1 + Z_2}{Z_1} - \frac{Z_1 + Z_2 + Z_3}{Z_1 Z_3} \cdot Z_2 = \frac{Z_1 Z_3}{Z_1 Z_3} = \underline{1}$ ✓

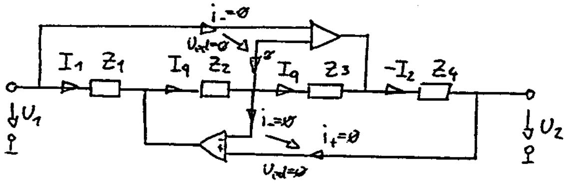
eg: Schaltung

es: Kettenmatrix $\underline{A}$
(ideale OPV's)



deale OPV's: $U_{ed} = \emptyset$; $i_+ = i_- = \emptyset$

chaltung:



$U_1 + 2U_{ed} = U_2 \Rightarrow U_1 = U_2$

$I_1 \cdot Z_1 + I_q \cdot Z_2 = \emptyset$

$I_q Z_3 - I_2 \cdot Z_4 = \emptyset \Rightarrow I_q = I_2 \cdot \frac{Z_4}{Z_3} \Rightarrow I_1 \cdot Z_1 + I_2 \cdot \frac{Z_4}{Z_3} \cdot Z_2 = \emptyset$

$I_1 = -\frac{Z_2 Z_4}{Z_1 Z_3} \cdot I_2$

$\Rightarrow \quad U_1 = A_{11} U_2 - A_{12} I_2$

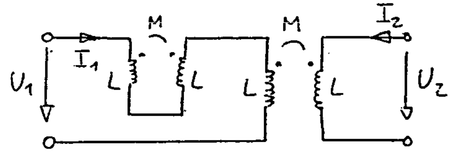
$I_1 = A_{21} U_2 - A_{22} I_2$

$$\Rightarrow \underline{A} = \begin{pmatrix} 1 & \emptyset \\ \emptyset & \frac{Z_2 Z_4}{Z_1 Z_3} \end{pmatrix}$$

FET

8.10) Geg: Zweitor

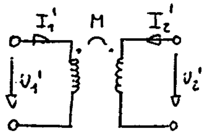
Ges: $\underline{Z}$



FET

$\underline{U} = \underline{Z} \cdot \underline{I} \Rightarrow U_1 = Z_{11} \cdot I_1 + Z_{12} I_2$

$U_2 = Z_{21} I_1 + Z_{22} I_2$



$U_1' = sLI_1' + sMI_2'$

$U_2' = sLI_2' + sMI_1'$

$Z_{11} = \frac{U_1}{I_1}\Big|_{I_2=\emptyset} \Rightarrow I_1 \cdot 3sL - I_1 \cdot sM - I_1 sM = U_1 \Rightarrow \underline{Z_{11} = 3sL - 2sM}$

$Z_{22} = \frac{U_2}{I_2}\Big|_{I_1=\emptyset} \Rightarrow U_2 = sL\, I_2 \Rightarrow \underline{Z_{22} = sL}$

$Z_{12} = \frac{U_1}{I_2}\Big|_{I_1=\emptyset} \Rightarrow U_1 = sM\, I_2 \Rightarrow \underline{Z_{12} = sM}$

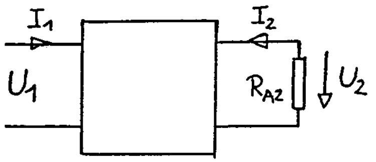
$Z_{21} = \frac{U_2}{I_1}\Big|_{I_2=\emptyset} \Rightarrow U_2 = sM\, I_1 \Rightarrow \underline{Z_{21} = sM = Z_{12}}$ passiver Vierpol

$$\underline{Z} = \begin{pmatrix} 3sL - 2sM & sM \\ sM & sL \end{pmatrix}$$

...g: Ein reziproker, passiver Vierpol wird mit der Sinusspannung von $U_{eff}=10V$ gespeist und mit $R_{A2}=100\Omega$ abgeschlossen.

Daten des Vierpols: $Z_{11}=20\Omega$; $Z_{22}=40e^{j45°}\Omega$; $Z_{12}=20\Omega\cdot e^{j60°}$

...es: kompl. $\underline{Z}$; Leistung P im Abschluß R_{A2}



① $U_1=Z_{11}\cdot I_1+Z_{12}I_2$

② $U_2=Z_{21}\cdot I_1+Z_{22}I_2$

③ $U_2=-R_{A2}\cdot I_2$; $P=R_{A2}\cdot I_2^2$ ④

$\rightarrow$ Ges: $I_2=f(U_1)$

②,③

$\Rightarrow -R_{A2}I_2=Z_{21}I_1+Z_{22}I_2 \Rightarrow I_1=\frac{1}{Z_{21}}\cdot(-R_{A2}-Z_{22})I_2$

①②,③

$$U_1=Z_{11}\cdot\frac{1}{Z_{21}}(-R_{A2}-Z_{22})\cdot I_2+Z_{12}I_2=I_2\cdot\left(Z_{12}-\frac{Z_{11}}{Z_{21}}(R_{A2}+Z_{22})\right)$$

$$I_2=\frac{U_1}{Z_{12}-\frac{Z_{11}}{Z_{21}}(R_{A2}+Z_{22})}$$

$$P=R_{A2}\cdot I_2^2=\frac{R_{A2}}{(Z_{12}-\frac{Z_{11}}{Z_{21}}(R_{A2}+Z_{22}))^2}\cdot U_1^2$$

$$=\begin{pmatrix}20 & 20e^{j60}\\ 20e^{j60} & 40e^{j45}\end{pmatrix}\Omega \qquad \text{reziprok} \Rightarrow Z_{12}=Z_{21}$$

$= \sqrt{2}\cdot U_{eff}\cdot\cos\omega t \qquad U_1^2=2\cdot U_{eff}^2\cdot(\frac{1}{2}+\frac{1}{2}\cos 2\omega t) \qquad \overline{U_1^2}=U_{eff}^2$

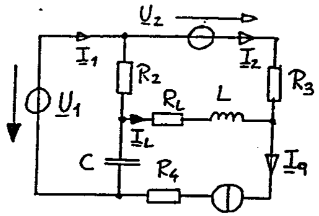
$$\frac{R_{A2}\cdot\overline{U_1^2}}{(Z_{12}-\frac{Z_{11}}{Z_{21}}(R_{A2}+Z_{22}))^2}=\frac{100}{(20e^{j60}-\frac{20}{20e^{j60}}\cdot(100+40e^{j45}))^2}\cdot\frac{\overline{U_1^2}}{\Omega}=\frac{100\cdot100}{|(20e^{j60}-100e^{-j60}-40e^{-j15})^2|}$$

$$\frac{10\,000}{|(10+j17{,}3-50+j86{,}6-38{,}6+j10{,}35)^2|}=\frac{10\,000}{|(-78{,}6+j114)^2|}=\frac{10\,000}{19\,242}=\underline{\underline{0{,}52\,W}}$$

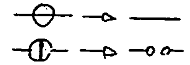
8.16) Geg: •) Brücke

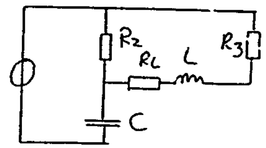
•) eingeschwungener Zustand $U=U_0\cdot\cos\omega t$

Ges: ΔI_L für ΔU_1 (Überlagerungssatz)



$\underline{I}_L=\underline{I}_L(U_1)+\underline{I}_L(U_2)+\underline{I}_L(I_q)$





$$\underline{I}_L(\underline{U}_1)=\underline{I}_1\cdot\frac{R_2}{R_2+R_3+R_L+j\omega L}$$

$$\underline{I}_1=\frac{\underline{U}_1}{\frac{1}{j\omega C}+R_2/\!/(R_3+R_L+j\omega L)}$$

$$\underline{I}_L(\underline{U}_1)=\frac{\underline{U}_1\cdot R_2}{(\frac{1}{j\omega C}+R_2/\!/(R_3+R_L+j\omega L))\cdot(R_2+R_3+R_L+j\omega L)}=G(j\omega)\cdot\underline{U}_1$$

$\underline{U}_1=\hat{U}_1\cdot\cos(\omega t)\rightarrow \underline{I}_L(\underline{U}_1)=\hat{I}_L\cdot\cos(\omega t+\varphi_G)=\hat{U}_1\cdot Re\{G(j\omega)\cdot e^{j\omega t}\}$

$\underline{U}_1'=(\hat{U}_1+\Delta U)\cdot\cos(\omega t)\rightarrow \underline{I}_L'(\underline{U}_1')=\underline{I}_L(\underline{U}_1)+\Delta\underline{I}=(\hat{U}_1+\Delta U)\cdot Re\{G(j\omega)e^{j\omega t}\}$

$\Rightarrow \Delta I\cdot Re\{e^{j\omega t}\}=\Delta\underline{I}=\Delta U\cdot Re\{G(j\omega)e^{j\omega t}\}$

$\underline{\Delta\underline{I}}=\Delta U\cdot|G(j\omega)|\cdot Re\{e^{j\omega t+j\varphi_G}\}=\underline{\Delta U\cdot|G(j\omega)|\cdot\cos(\omega t+\varphi_G)}$

$$G(j\omega)=\frac{R_2}{(\frac{1}{j\omega C}+R_2/\!/(R_3+R_L+j\omega L))\cdot(R_2+R_3+R_L+j\omega L)}$$